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Enhancing Fuzzy C-Means Performance for Low Fuzzifier Values through Ordered Weighted Aggregation

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Abstract

A prominent fuzzy clustering technique is the Fuzzy C-Means (FCM) in which the fuzzifier value $\boldsymbol{m}$ has a substantial effect on the algorithm's performance. Standard FCM may become unstable in this environment even though smaller values of $\boldsymbol{m}$ result in more refined partitions. Ordered Weighted Aggregation Fuzzy C-Means (OWA-FCM), which integrates an Ordered Weighted Aggregation (OWA) operator into the membership update stage, is our approach to address this problem. An orness parameter($\boldsymbol{p_0}$) controls the binomial distribution which is used to calculate the OWA operator's weights. For various values of $\boldsymbol{m}$ and $\boldsymbol{p_0}$, extensive experimentation on benchmark datasets were performed. Obtained results clearly indicate that, for $\boldsymbol{m \gtrsim 2.0}$, OWA-FCM behaves as regular FCM. On the other hand, OWA-FCM generates more accurate and consistent clustering for lower fuzzifier values ($\boldsymbol{m \lesssim 1.8}$), particularly $\boldsymbol{m \leq 1.4}$, leading to better ARI scores and crisper partitions. In addition, throughout a broad range of orness values ($\boldsymbol{0.001 \leq p_0 \leq 0.999}$), the final clustering results remain unchanged, suggesting that the OWA-based aggregating mechanism, rather than the particular selection of $\boldsymbol{p_0}$, is accountable for the improvement. These results indicate that when more precise cluster assignments are required, OWA-FCM is a reliable substitute for conventional, standard FCM.

Keywords: Clustering, Fuzzy C-Means, Ordered Weighted Aggregation Operators, Binomial OWA Operator, Membership, Fuzziness, Orness.

1 Introduction

Clustering is a key component of unsupervised machine learning, tasked with partitioning a dataset into groups, or clusters, such that elements within a cluster are more similar to one another than to those in other clusters [1–4]. This task is fundamental to discovering underlying structures and insights across a variety of data domains [5]. Among the various available clustering algorithms, Fuzzy C-Means (FCM) [6, 7] stands out as a prominent

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method. It is valued for its ability to assign partial memberships of data points to multiple clusters, thereby offering greater effectiveness in handling overlapping or ambiguously defined cluster boundaries [8, 9].

Let $\boldsymbol{X} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$ be a dataset of N data points in $\mathbb{R}^D$. Standard FCM partitions $\boldsymbol{X}$ into C fuzzy clusters by minimizing the objective function:

$$J_m(\mathbf{U}, \mathbf{V}) = \sum_{k=1}^{N} \sum_{i=1}^{C} (u_{ik})^m ||\boldsymbol{x}_k - \boldsymbol{v}_i||^2 \tag{1}$$

where $\mathbf{U} = [u_{ik}]$ is the fuzzy partition matrix ($u_{ik} \in [0, 1]$, $\sum_{i=1}^{C} u_{ik} = 1$), $\mathbf{V} = \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_C\}$ are the cluster centroids ($\boldsymbol{v}_i \in \mathbb{R}^D$), and $m \in (1, \infty)$ is the fuzzifier. The parameter m controls the level of fuzziness of the partition. As $m \to 1^+$, FCM tends towards a hard partition, while larger m values increase fuzziness. Minimization of J_m involves iterative updates for u_{ik} and $\boldsymbol{v}_i$ [10].

Despite its utility, standard FCM has limitations. The global fuzzifier m uniformly affects all clusters, which may not be ideal for complex datasets with varying cluster characteristics [1]. Furthermore, when m is set to low values (approaching 1) to achieve crisper partitions, standard FCM can become unstable or yield poor clustering results for certain datasets, as its membership update mechanism becomes highly sensitive to small distance variations.

Ordered Weighted Aggregation (OWA) operators, introduced by Yager [11], offer a flexible class of aggregation functions. An OWA operator of dimension p maps $\mathbb{R}^p \to \mathbb{R}$ using a weighting vector $\boldsymbol{W} = (w_1, \ldots, w_p)^T$ (where $w_j \in [0, 1]$, $\sum w_j = 1$) applied to arguments sorted in descending order (b_j):

$$\mathrm{OWA}_{\boldsymbol{W}}(a_1, \ldots, a_p) = \sum_{j=1}^{p} w_j b_j \tag{2}$$

The behavior of an OWA operator can be characterized by its 'orness' measure [11, 12],

$$\mathrm{orness}(\boldsymbol{W}) = \sum_{j=1}^{p} w_j \frac{p-j}{p-1} \tag{3}$$

which ranges from 0 (min-like) to 1 (max-like). OWA weights can be generated to achieve a specific target 'orness', for instance, using binomial distributions [13–15].

This paper introduces a modification to the Fuzzy C-Means algorithm, termed OWA-FCM (Ordered Weighted Aggregation FCM). The key innovation is the integration of an OWA operator into the membership update step. The standard FCM membership calculation involves an inverse sum of distance-ratio terms. OWA-FCM replaces this simple summation with a more structured OWA aggregation. For this aggregation, we use a set of theoretically grounded weights generated via a binomial distribution, which is parameterized by a target 'orness' level, p_0. The primary goal is to investigate whether this structural modification can improve FCM's performance and stability, especially at low fuzzifier (m) values where conventional FCM may struggle. As our results will show, while p_0 is used to generate the weights, the final performance of OWA-FCM is surprisingly robust and insensitive to the choice of p_0, indicating the structural change itself is the key contribution.

The main contributions of this work are:

1. We propose OWA-FCM, a new FCM variant that incorporates an OWA operator into the membership update process.
2. We show experimentally that OWA-FCM provides more accurate and stable clustering than standard FCM for low fuzzifier values ($m < 2.0$), predominantly when $m \leq 1.4$.
3. We demonstrate that the final clustering results are insensitive to the orness parameter p_0 used to generate the binomial OWA weights, making the method easier to apply in practice.
4. We observe that OWA-FCM converges to standard FCM behavior for $m \gtrsim 2.0$, instituting a clear operating range in which the proposed method offers the greatest benefit.

The results obtained in this work offer new insights into the behavior of FCM-type algorithms, particularly demonstrating how modifying core aggregation mechanisms may lead to enhanced robustness.

Section 2 establishes the theoretical frameworks and reviews related work. The OWA-FCM algorithm is developed in Section 3. In Section 4, a rigorous experimental setup is presented, with results and discussion in detail. Section 5 concludes the paper.

2 Literature Review

For fuzzy clustering ,the FCM algorithm [6] is a rudimentary method, but limitations posed by the FCM algorithm have led extensive research on modified algorithm. In FCM algorithm control mechanism provided by the global fuzziness parameter (m). Application of this control mechanism uniformly across all clusters, is a major drawback [16, 17]. Choosing an improper value of m results in poorly defined clusters or overly diffused memberships [18]. Additionally, standard FCM shows instability or poor performance on certain datasets while choosing very low values (approaching 1) of m to realize crisper partitions.
Several modified FCM algorithms have been proposed to address such challenges. These can be broadly categorized. In the first category, the objective function or the distance metric modified, for instance, Possibilistic Fuzzy C-Means (PFCM) [19] and Fuzzy C-Medoids (FCMdd) [20]. In second category, the distance metric adapted by using a fuzzy covariance matrix to handle non-spherical cluster shapes e.g. Gustafson-Kessel (GK) algorithm [21]. In another method like Kernel FCM (KFCM) [22], where data is mapped into higher-dimensional spaces by employing kernel functions for non-linear boundary detection.

Another major line of research focuses on adapting the fuzziness parameter itself. This incorporates techniques like estimating an optimal global m [23], proposing cluster-wise fuzzifiers (m_i) [24], and even point-wise fuzziness parameters [25]. Entropy-regularized FCM where the entropy term added to the objective function results in alteration of membership sharpness [26]. Recently, researchers across the globe have been working to address limitations posed in FCM algorithm, for instance, in robust image segmentation adaptively integrates spatial information [27] or by using self-adaptive fuzzy exponents in medical imaging [28].

Significant progress have been made in the field of variants of FCM algorithm. These variant FCM mainly focus on modification of the objective function, the distance metric, or the fuzziness parameter. In this manuscript, we propose OWA-FCM algorithm in which internal mechanism of the membership update rule itself gets modified. In this algorithm summation of distance-ratio terms is replaced with OWA aggregation, which is more sophisticated than before. This modification in aggregation process directly influences the parameter m (particularly low value of m), may prove to be potentially powerful algorithm to address the limitations.

2.1 Ordered Weighted Aggregation (OWA) Operators

The notion of OWA operators was proposed by Yager [11], which represents an aggregation function belongs to parameterized class. For employing OWA , a two-step process is applied. In the first step, the arguments, which are to be aggregated, sorted either in descending or ascending order. In the second step, the weighting vector $\boldsymbol{W}$ (as in Eq. 2) is applied to the arranged arguments. This allows aggregation weights to correspond to an argument's value-based rank, offering considerable flexibility in modeling aggregation behavior. The characteristics of operator behavior are estimated by 'orness' measure. The value of 'orness' ranges from min-like ('orness'=0) to max-like ('orness'=1).

However, determining the OWA weights $\boldsymbol{W}$ is vital step for its application. For the condition requiring a particular aggregation attitude, methods to obtain weights that satisfy a predefined 'orness' level are indispensable. Yager and others have devised several techniques [29]. Specifically, Binomial distribution, a well established method is used for generating OWA–weights. These weights yields yield a precise target 'orness' p_0 is a well-established method [30]. Given C arguments, the weights w_j (for $j = 1, \ldots, C$) associated with the j-th largest argument can be derived using a binomial scheme designed to exhibit the target 'orness' p_0. For designing the OWA component of our proposed algorithm this technique is utilized.

OWA operators have been employed in various ares such as multi-criteria decision making [31], data mining, and various fuzzy systems [32]. Futhermore, in case of data analysis, this technique applied in areas like classifier output aggregation [33] and feature evaluation [34] in data analysis. Recent development showed that this can also be employed in areas like group decision-making with hesitant fuzzy linguistic term sets [35] and in frameworks for sustainable supplier selection [36].

While OWA operators are widely studied, this work investigates their direct integration into the iterative membership update of FCM. In the proposed OWA-FCM algorithm, we employ binomial OWA weights (parameterized by p_0) to aggregate the distance-ratio terms. A key focus of our study is to show how this structural change, rather than the fine-tuning of p_0, provides a performance advantage over standard FCM, especially in the challenging low-fuzzifier regime.

3 Proposed Method: OWA-FCM

In this section, we introduce the Ordered Weighted Aggregation Fuzzy C-Means (OWA-FCM) algorithm. The rationale for integrating the OWA operator into the FCM membership update step is discussed.

The standard FCM algorithm determines the membership u_{ik} of a data point $\boldsymbol{x}_k$ to cluster i based on the formula: $u_{ik} = \left(\sum_{j=1}^{C}(d_{ik}/d_{jk})^{\frac{2}{m-1}}\right)^{-1}$, where $d_{ik} = ||\boldsymbol{x}_k - \boldsymbol{v}_i||$. The terms $T_{ikj} = (d_{ik}/d_{jk})^{\frac{2}{m-1}}$ represent the relative proximity of $\boldsymbol{x}_k$ to the i-th cluster compared to the j-th cluster, scaled by the fuzzifier m. Standard FCM uses a simple summation of these T_{ikj} terms in the denominator.

The basic premise of OWA-FCM is to use a more structured OWA operator in place of this straightforward summation. The objective is to explore if changing this basic aggregation step can result in different, possibly advantageous clustering behaviors, especially with regard to partition quality and stability when low fuzzifier values (m) are used—a domain where typical FCM can suffer. To build the OWA operator, we require a $\boldsymbol{W}_{\text{mem}} = (w_1, \ldots, w_C)^T$. These weights are computed in this work using a binomial distribution technique, a popular method parameterized by a desired orness level $p_0 \in [0, 1]$ [37]. In contrast, our comprehensive empirical results (displayed in Section 4) algorithm: for this specific FCM modification, the choice of p_0 has no impact on the final clustering result.

Given this, The reason for OWA-FCM shifts from using considering the impact of the OWA aggregate structure itself. Unlike the direct sum of FCM, this structure applies a set of normalized, non-uniform weights after sorting the T_{ikj} elements. This structural variance seems to be the primary cause of the observed behavioral shifts, specifically the enhanced performance and stability of OWA-FCM compared to standard FCM when $m < 2.0$. The OWA mechanism offers an alternate method of processing the distance-ratio evidence, which is beneficial under low-m conditions, even if the overall membership calculation neutralizes its 'orness'-specific tuning via p_0.

3.1 Algorithm Derivation

Let the dataset be $\boldsymbol{X} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$, with $\boldsymbol{x}_k \in \mathbb{R}^D$. We aim to partition $\boldsymbol{X}$ into C clusters with centroids $\mathbf{V} = \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_C\}$. The fuzzifier is $m > 1$. For the OWA operator, weights are generated based on a target 'orness' $p_0 \in [0, 1]$ to ensure a theoretically grounded weight distribution, although, as noted, the final outcome of OWA-FCM is found to be insensitive to p_0.

The standard FCM membership update is given by:

$$u_{ik} = \left(\sum_{j=1}^{C} \left(\frac{||\boldsymbol{x}_k - \boldsymbol{v}_i||}{||\boldsymbol{x}_k - \boldsymbol{v}_j||} \right)^{\frac{2}{m-1}} \right)^{-1} \tag{4}$$

Let $d_{sk} = ||\boldsymbol{x}_k - \boldsymbol{v}_s||$. The terms being aggregated are $T_{ikj} = (d_{ik}/d_{jk})^{\frac{2}{m-1}}$.

In OWA-FCM, the aggregation of these T_{ikj} terms is modified as follows:

1. For each data point $\boldsymbol{x}_k$ and for each cluster i: Calculate the set of C terms $\mathcal{T}_{ik} = \{T_{ik1}, T_{ik2}, \ldots, T_{ikC}\}$.
2. Sort the terms in $\mathcal{T}_{ik}$ in descending order: $b_{ik,1} \geq b_{ik,2} \geq \ldots \geq b_{ik,C}$.
3. Generate the OWA weighting vector $\boldsymbol{W}_{\text{mem}} = (w_1, \ldots, w_C)^T$. The weights w_l (for $l = 1, \ldots, C$), associated with $b_{ik,l}$, are determined using the binomial formula based on the target 'orness' p_0:

$$w_l = \binom{C-1}{l-1} (1 - p_0)^{l-1} p_0^{C-l}, \quad l = 1, \ldots, C. \tag{5}$$

These weights satisfy $\sum_{l=1}^{C} w_l = 1$.

4. The OWA-aggregated sum for point $\boldsymbol{x}_k$ and cluster i is:

$$S_{ik}^{\text{OWA}} = \sum_{l=1}^{C} w_l b_{ik,l} \tag{6}$$

5. The unnormalized membership u'_{ik} is the inverse of this sum:

$$u'_{ik} = (S_{ik}^{\text{OWA}})^{-1} \tag{7}$$

   Numerical stability requires handling cases where S_{ik}^{OWA} is near zero.
6. Memberships are normalized across all clusters for the given data point:

$$u_{ik} = \frac{u'_{ik}}{\sum_{s=1}^{C} u'_{sk}} = \frac{(S_{ik}^{\text{OWA}})^{-1}}{\sum_{s=1}^{C} (S_{sk}^{\text{OWA}})^{-1}} \tag{8}$$

Special Case Handling for $d_{ik} = 0$: If a data point $\boldsymbol{x}_k$ coincides with a centroid $\boldsymbol{v}_i$ ($d_{ik} = 0$), then its membership is crisply assigned: $u_{ik} = 1$ and $u_{sk} = 0$ for all $s \neq i$. This is handled before the OWA aggregation. If $d_{ik} \neq 0$ but $d_{jk} = 0$ for some $j \neq i$, then $T_{ikj} \to \infty$. With $w_1 > 0$ (for $p_0 < 1$), $S_{ik}^{\text{OWA}} \to \infty$, leading to $u'_{ik} \to 0$, as expected.

The centroid update rule remains identical to standard FCM:

$$\boldsymbol{v}_i = \frac{\sum_{k=1}^{N} (u_{ik})^m \boldsymbol{x}_k}{\sum_{k=1}^{N} (u_{ik})^m} \tag{9}$$

OWA-FCM iterates between computing memberships u_{ik} (Eqs. 5-8) and updating centroids $\boldsymbol{v}_i$ (Eq. 9) until convergence.

3.2 Algorithm Steps

The iterative procedure for OWA-FCM is summarized in Algorithm 1.

Note: δ_{zero}, δ_{num}, δ_{sum} are small positive constants for numerical stability. $||\cdot||_F$ denotes the Frobenius norm.

3.3 Computational Complexity

The standard FCM algorithm has a per-iteration complexity of approximately $O(NCD)$, dominated by distance calculations and centroid updates. For OWA-FCM, the membership update for each of the N data points involves an additional overhead for each of the C prospective cluster memberships:

- Calculating C distance-ratio terms (T_{ikj}): $O(C)$ per membership calculation.
- Sorting these C terms: $O(C \log C)$.
- Performing the OWA aggregation (weighted sum): $O(C)$.

This process is repeated for each of the $N \times C$ memberships to be calculated. The total complexity per iteration is therefore dominated by the initial distance calculations ($O(NCD)$), the sorting step ($O(NC \cdot C \log C)$), and the centroid updates ($O(NCD)$). The overall complexity per iteration is approximately $O(NCD + NC^2 \log C)$. Compared to standard FCM's $O(NCD)$, OWA-FCM introduces an additional term that depends polynomially

Algorithm 1 Proposed OWA-FCM Algorithm

Input: Dataset $\boldsymbol{X} = \{\boldsymbol{x}_1, \dots, \boldsymbol{x}_N\}$; Number of clusters C; Fuzzifier $m > 1$; Parameter $p_0 \in [0, 1]$ for OWA weight generation; Convergence threshold $\epsilon > 0$; Maximum iterations $MaxIter$.
Output: Fuzzy partition matrix $\mathbf{U} = [u_{ik}]$; Cluster centroids $\mathbf{V} = \{\boldsymbol{v}_1, \dots, \boldsymbol{v}_C\}$.
Initialize cluster centroids $\mathbf{V}^{(0)} = \{\boldsymbol{v}_1^{(0)}, \dots, \boldsymbol{v}_C^{(0)}\}$ (e.g., randomly).
Generate OWA weights $\boldsymbol{W}_{\text{mem}} = (w_1, \dots, w_C)^T$ using Eq. 5 with parameter p_0.
$t \leftarrow 0$.
repeat
    $t \leftarrow t + 1$.
    Update Membership Matrix $\mathbf{U}^{(t)}$:
    for all data points $\boldsymbol{x}_k \in \boldsymbol{X}$ **do**
        if any $||\boldsymbol{x}_k - \boldsymbol{v}_s^{(t-1)}|| < \delta_{zero}$ for some s **then**
            Set $u_{sk}^{(t)} \leftarrow 1$ for the coincident cluster and $u_{jk}^{(t)} \leftarrow 0$ for others.
            continue to next data point.
        end if
        for all clusters $i = 1, \dots, C$ **do**
            Let $d_{ik} = ||\boldsymbol{x}_k - \boldsymbol{v}_i^{(t-1)}||$.
            Compute terms $T_{ikj} = \left(d_{ik}/(||\boldsymbol{x}_k - \boldsymbol{v}_j^{(t-1)}|| + \delta_{num})\right)^{\frac{2}{m-1}}$ for $j = 1, \dots, C$.
            Sort $\{T_{ikj}\}_{j=1}^{C}$ in descending order to get $b_{ik,1} \geq \ldots \geq b_{ik,C}$.
            $S_{ik}^{\text{OWA}} \leftarrow \sum_{l=1}^{C} w_l \cdot b_{ik,l}$.
            $u'_{ik} \leftarrow (S_{ik}^{\text{OWA}} + \delta_{sum})^{-1}$.
        end for
        Normalize: $u_{ik}^{(t)} \leftarrow u'_{ik} / \sum_{s=1}^{C} u'_{sk}$ for all $i = 1, \dots, C$.
    end for
    Update Centroids $\mathbf{V}^{(t)}$:
    for all clusters $i = 1, \dots, C$ **do**
        num $\leftarrow \sum_{k=1}^{N} (u_{ik}^{(t)})^m \boldsymbol{x}_k$.
        den $\leftarrow \sum_{k=1}^{N} (u_{ik}^{(t)})^m$.
        $\boldsymbol{v}_i^{(t)} \leftarrow \text{num}/(\text{den} + \delta_{sum})$.
    end for
until $||\mathbf{V}^{(t)} - \mathbf{V}^{(t-1)}||_F < \epsilon$ or $t \geq MaxIter$.
Return $\mathbf{U}^{(t)}$ and $\mathbf{V}^{(t)}$.

on the number of clusters, C. For small C, this overhead is modest, but it becomes more significant for problems with a large number of clusters.

4 Experimental Framework

This section presents details the experimental framework established to evaluate the proposed OWA-FCM algorithm. It also describes the datasets to be used, the algorithms chosen for comparison analysis, the particular parameter values applied here, and the performance evaluation measures employed in the experimentation setup.

Table 1: Summary of Datasets Used in Experiments.

Dataset Name	# Features (D)	# Samples (N)	# Classes (C)
Iris	4	150	3
Seeds	7	210	3
User Knowledge Modeling	12	182	2
Wine	13	178	3
Breast Cancer Wisconsin	30	569	2
Ionosphere	34	351	2

4.1 Experimental Protocols

4.1.1 Datasets

Six publicly accessible datasets to evaluate from the UCI Machine Learning Repository were employed here. The number of samples (N), features (D), and classes (C) in these datasets vary. Table 1 provides an outline. All features were normalized to have a zero mean and unit variance before clustering.

4.1.2 Algorithms Used for Comparison

OWA-FCM's performance is compared with standard FCM [10] and K-Means [38]. To generate the binomial OWA weights for OWA-FCM and examine the effect of the orness parameter, we examined a sample range of p_0 values $0.001, 0.5, 0.999$.

4.1.3 Parameter Settings

In order to match the actual number of classes in each dataset, the number of clusters (C) were chosen. Experiments were carried out for $m \in -1.1, 1.2, 1.3, 1.4, 1.5, 1.75, 2.0, 2.5, 3.0$ to explore the impact of fuzziness. When the maximum number of iterations (100) was reached or the Frobenius norm of the centroid update dropped below $\epsilon = 10^{-5}$, the algorithms were deemed to have converged. Using the same randomly initialized centroids for every algorithm in a given run, each approach was run ten times for a fair comparison. The evaluation metrics mean and standard deviation matches the stated values.

4.1.4 Performance Measures

Five popular measures were employed to evaluate clustering performance: Adjusted Rand Index (ARI) [39], Normalized Mutual Information (NMI) [40], Fuzzy Partition Coefficient (FPC) [6], Partition Entropy (PE) [6], and Silhouette Score [41]. While FPC and PE assess the level of fuzziness in the partition, ARI and NMI gauge agreement with the true class labels. Cluster cohesion and separation are evaluated here using the Silhouette Score. ARI, NMI, and Silhouette score were calculated using the crisp partition achieved by allocating each data point to the cluster with greatest membership value for fuzzy clustering techniques (FCM and OWA-FCM).

4.2 Results and Discussion

To address the main issues of the research, the experimental results are examined here in this section highlighting the main key findings and offers illustrative examples.

4.2.1 RQ1: Behavior of OWA-FCM vs. Standard FCM across m values

For $m \gtrsim 1.8$, and specifically for $m \geq 2.0$ (a common default value), OWA-FCM consistently generates results that are identical to standard FCM across all datasets used and metrics applied. This suggests a practical and likely mathematical equivalence under these conditions.

On the other hand, for $m \lesssim 1.8$, notable and significant differences emerge. At very low m values (e.g., $m = 1.1, 1.2$), standard FCM often exhibited instability or poor performance. For example, on the Wine dataset with $m = 1.1$, FCM achieved a mean ARI near zero, indicating a random partitioning. In stark contrast, OWA-FCM achieved a robust mean ARI of approximately 0.90. Similar significant improvements for OWA-FCM were observed on the Breast Cancer and Ionosphere datasets under these low-m conditions. This indicates that the OWA aggregation provides a stabilizing effect, allowing the algorithm to find meaningful structures even when enforcing very crisp partitions. As m increases towards 2.0, the performance of the two algorithms converges.

Regarding partition characteristics, for $m < 2.0$, OWA-FCM consistently produced partitions that were as crisp or crisper (higher FPC, lower PE) than standard FCM, while maintaining or improving partition quality (ARI). For example, on the Wine dataset with $m = 1.1$, FCM produced a nearly random, maximally fuzzy partition (FPC ≈ 0.35, PE ≈ 0.98), while OWA-FCM maintained high crispness (FPC ≈ 0.96, PE ≈ 0.06) along with its high ARI.

4.2.2 RQ2: Impact of the 'Orness' parameter p_0 on OWA-FCM

One significant and rather unusual outcome is that OWA-FCM is unaffected by the selection of the 'orness' parameter p_0 when employing the binomial weight generation scheme. All tested OWA-FCM versions (from $p_0 = 0.001$ to $p_0 = 0.999$) converged to the exact same final centroids and partition matrix for any given dataset and fixed fuzzifier m, resulting in identical values for all metrics of performance.

This substantially implies that the specific distribution of those binomial weights (as determined by p_0) does not ultimately change the final partition, even though the OWA operator's structure (ordering the distance-ratio terms and applying normalized weights) allows it to behave differently from FCM's simple sum. The deviation from typical FCM is mostly caused by the ordered, weighted aggregating process itself, not by the operator's particular "orness" in this situation. Because OWA-FCM does not introduce a new, sensitive hyperparameter that needs to be adjusted each time, this is a highly desirable characteristic.

4.2.3 RQ3: Conditional Equivalence with Standard FCM

Results observed here show that for $m \gtrsim 1.8$, OWA-FCM with binomial weights, it is similar to normal FCM, and for all investigated scenarios, perfect agreement was observed when $m \geq 2.0$. External metrics like ARI and NMI frequently stay the same in the range $1.5 \leq m < 1.8$, however internal metrics like FPC and PE indicate that OWA-FCM creates somewhat crisper partitions before converging to typical FCM behavior. Thus, at lower fuzziness levels ($m < 2.0$), OWA-FCM is extremely beneficial to create sharper clusters; as m increases, it naturally decreases compared to the standard/regular FCM.

4.2.4 RQ4: Qualitative Differences in Membership Structure

The numerical results that are obtained, indicate that the membership structures created at low m values differ considerably. OWA-FCM generates more precise and structurally accurate membership assignments, according to its higher ARI scores in cases, where normal FCM fails (e.g., $m = 1.1$). The much higher FPC and lower PE values corroborate this, indicating that OWA-FCM avoids collapsing into the highly ambiguous or unstable states, that standard FCM can fall into when pushed to be overly crisp. The insensitivity of OWA-FCM's final metrics to p_0 further implies that, for a given m, the membership matrix $\mathbf{U}$ it converges to is the same regardless of the p_0 used for binomial weight generation.

4.2.5 Theoretical Discussion on Observed Behavior of OWA-FCM

Insensitivity to p_0***:***

In the normalizing stage of the membership update (Eq. 8) is likely the root cause of the final partition's observed insensitivity to p_0. The membership u_{ik} is computed as follows: $u_{ik} = (S_{ik}^{\mathrm{OWA}})^{-1} / \sum_{s=1}^{C} (S_{sk}^{\mathrm{OWA}})^{-1}$. Although varying p_0 modifies the OWA weights w_l and, therefore, the amount of the aggregated sum S_{ik}^{OWA}, it seems to do so in a proportionate way across all clusters for a particular data point. If changing p_0 results in a new aggregated sum $S'_{ik} \approx \alpha_k S_{ik}$ for some scaling factor α_k that depends on the data point $\boldsymbol{x}_k$ but not on the cluster i, then this factor would cancel out in the ratio: $u_{ik} \propto (\alpha_k S_{ik})^{-1} / \sum_s (\alpha_k S_{sk})^{-1} = S_{ik}^{-1} / \sum_s S_{sk}^{-1}$. This would lead to an identical membership matrix $\mathbf{U}$ regardless of p_0, which is consistent with our findings. A formal proof of this property is a subject for future research.

Equivalence with FCM for $m \geq 2.0$***:***

An analysis of the terms being aggregated, $T_{ikj} = (d_{ik}/d_{jk})^{2/(m-1)}$, clarifies the reason that why OWA-FCM converges to standard FCM for increasing values of m. The exponent $2/(m-1)$ declines and gets closer to zero as m increases and is zero for $m \to \infty$. The ratios are 'compressed' and brought near or closer to 1.0 by a reduced positive exponent. The collection of terms $T_{ikj}\}_{j=1}^{C}$ thus becomes highly concentrated with low variance for large m. The sorting effect becomes marginal when the arguments to an OWA operator are quite similar, and the OWA aggregation acts similarly to a straightforward arithmetic mean and a simple sum is the typical FCM denominator. This provides sufficient clarification on the equivalency that was shown in our results.

4.2.6 Computational Performance

As stated in Section 3.3, OWA-FCM's per-iteration complexity is $O(NCD + NC^2 \log C)$ versus normal FCM's $O(NCD)$. The additional $O(NC^2 \log C)$ term results from the cost of sorting C distance-ratio terms for every $N \times C$ membership computations. This suggests that OWA-FCM has higher computing costs but for datasets with few clusters (such as $C = 2$ or $C = 3$, as we see in our trials), the additional cost is somewhat negligible. However, this quadratic dependence on C would represent a significant obstacle for applications that need a large number of clusters (e.g., $C > 50$). As a result, we can observe that there is a trade-off between OWA-FCM's improved robustness at low m values and its higher computational cost.

5 Conclusion

OWA-FCM, a novel variant of a conventional Fuzzy C-Means that integrates an Ordered Weighted Aggregation operator into the membership update algorithm, was presented in this investigation. We showed through extensive testing that this structural change results in an algorithm that performs differently from traditional FCM and is often more advantageous.

The primary result is the fact that when using low fuzzifier values ($m < 2.0$), OWA-FCM offers a more reliable and stable substitute for conventional FCM. OWA-FCM consistently generated crisp, high-quality partitions in this setting, when conventional FCM may deliver unstable or inferior results. OWA-FCM displays a conditional equivalency with normal FCM at higher fuzzifier levels ($m \geq 2.0$). The somewhat unexpected insensitivity of the algorithm's final clustering results to the 'orness' parameter p_0 that is used in generating the binomial OWA weights was a second important finding. This may indicate that the OWA structure itself, rather than its 'orness'-based parameterization, is responsible for the performance gains. The above discussion simplifies the practical implementation of OWA by eliminating the requirement for an additional tuning parameter.

The focus on a single (binomial) method for OWA weight generation and the increased computing cost of OWA-FCM due to this and the sorting step, which may restrict its scalability for the situations having a very large number of clusters, can be classified as the two main limitations of this study. Our experiments were conducted on benchmark datasets of moderate size, validation on larger-scale datasets is needed to fully assess the algorithm's performance in big data contexts. Future work should explore alternative OWA weighting schemes such as hypergeometric and Stancu polynomial based schemes and investigate the theoretical underpinnings of the observed conditional equivalence and p_0 insensitivity with full mathematical rigor. Exploring algorithmic optimizations to reduce the computational overhead and applying OWA-FCM to a wider range of real-world problems and applications might be the promising research directions.

As per our analysis, OWA-FCM stands as a valuable modification to the classic FCM algorithm, offering enhanced performance for achieving crisp, stable partitions without introducing additional tuning complexity.

Declarations

- The authors received no specific funding for this study.

- The authors declare that they have no conflicts of interest to report regarding the present study.
- No Human subject or animals are involved in the research.
- All authors have mutually consented to participate.
- All the authors have consented the Journal to publish this paper.
- Authors declare that all the data being used in the design and production cum layout of the manuscript is declared in the manuscript.

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