Finite Time Thermodynamic Optimization of an Irreversible Heat Engine Coupled to Variable Temperature Reservoirs

Govind Maheshwari, Ashutosh Mehta, Sharad Chaudhary, Sunil K. Somani*

ABSTRACT

This paper presents the finite time thermodynamic optimization based on the maximum power density criterion for an irreversible Carnot heat engine model coupled to variable temperature heat reservoirs. Expressions for the power density and optimal efficiency at maximum power density conditions are derived by incorporating irreversibilities: finite rate heat transfer and internal irreversibility. The obtained results are compared with those obtained by using maximum power criterion. The design parameters under the optimal conditions have been derived analytically and the effect of the irreversibilities on the general and optimal performances are investigated. The results showed that the design parameters at maximum power density lead to smaller and more efficient heat engines. It is also seen that the irreversibilities have a greater influence on the maximum power density conditions in comparison to maximum power conditions of the heat engine. The results obtained herein generalize the results of previous studies on this subject and provide guidance to optimal design in terms of power, thermal efficiency and engine sizes for real heat engines.

Key-Words: - Finite time thermodynamics, maximum power density, Irreversible Carnot heat engine, Variable temperature reservoirs, Optimal design.

1. Introduction

Since finite time thermodynamics (FTT) or entropy generation minimization was advanced. much work has been carried out for the performance analysis and optimization of finite time processes and finite size devices [1-11]. In these studies, the power output, thermal efficiency, entropy generation and the ecological benefits are chosen for the optimization criteria. However, the performance analyses based on the above optimization criteria do not take the effects of engine sizes related to the investment cost into account. In order to include the effects of engine size in the performance analysis, Sahin et al introduced the maximum power density (MPD) as a new optimization criterion. Using the MPD criterion, they investigated optimal performance conditions for reversible[12] and irreversible[13] non regenerative Joule-Brayton heat engines. In their study, they maximized the power density (the ratio of power to the maximum specific volume in the cycle) and found design parameters at MPD conditions,

which led to smaller and more efficient Joule-Brayton heat engines than those engines working at maximum power(MP) conditions. The power density objective was also applied to an irreversible radiative heat engine[14], ideal reversible Ericsson cycle[15] and an Atkinson cycle[16] free of any irreversibility. Sahin et al[17] applied the MPD technique to the endoreversible Carnot heat engine which can be considered as a theoretical comparison standard for all real heat engines in finite time thermodynamics. In all the above references [12-17], researchers applied MPD techniques to the heat engines having thermal reservoirs of infinite heat capacity. However, for practical applications it is very important to investigate the performance of heat engine when the effect of finite thermal capacitance of thermal reservoirs is taken into account. Recently Chen et al [18] have applied MPD techniques to an endoreversible closed variable temperature heat reservoir Brayton cycle. The present work is different from a recent work of authors [19]. In reference [19], the

erformance of an endoreversible Carnot heat engine was analysed using the power density objective with considerations of the heat transfer irreversibility in the hot and cold side heat exchangers and the effects of finite thermal capacitance rates.

2. System Description and Analysis

A Heat engine working between two thermal reservoirs of finite heat capacitance is shown in Figure 1. The processes 1-2' and 3'-4 are reversible adiabatic, and processes 2'-3' and 4-

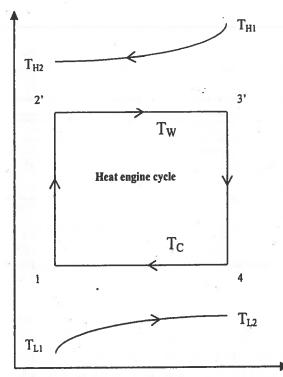


Figure 1 Schematic diagram of a Carnot heat engine

1 are isothermal. Assuming that the heat exchangers are counterflow, the heat conductance (product of heat transfer surface area and heat transfer coefficient) of the hot and cold side heat exchangers are U_H and U_L . The thermal capacity rate (product of mass flow rate and specific heat) of the hot side heat reservoir is C_H , and the inlet and outlet temperature of the heating fluid are T_{H1} and T_{H2} , respectively. The thermal capacity rate of the cold side heat reservoir is C_L , and the inlet and outlet temperatures of the cooling fluid are T_{L1} and T_{L2} , respectively. Temperatures of hot and cold side working fluid of the heat engine are T_W and T_C respectively. The T-S diagram of the heat engine

is shown in Figure 2.

The rate of heat flow (Q_H) from high temperature heat source to the system is given by:

$$Q_{H} = C_{H} (T_{H1} - T_{H2}) = \varepsilon_{H} C_{H} (T_{H1} - T_{W})$$
 (1)

Similarly, the rate of heat flow (Q_L) from system to the low temperature heat sink is

$$Q_L = C_L \left(T_{L1} - T_{L2} \right) = \varepsilon_L C_L \left(T_C - T_{L1} \right) \tag{2}$$

where e_H and e_L are, respectively, the effectiveness of the hot and cold side heat exchangers, defined as:

$$\varepsilon_H = 1 - \exp\left[-N_H\right] \tag{3}$$

$$\varepsilon_L = 1 - \exp\left[-N_L\right] \tag{4}$$

The number of heat transfer units, N_H and N_L are based on the minimum thermal capacitance rates, that is:

$$N_{H} = \frac{U_{H}}{C_{H}} \qquad N_{L} = \frac{U_{L}}{C_{L}} \tag{5}$$

The power produced (W) by the engine according to the first law is:

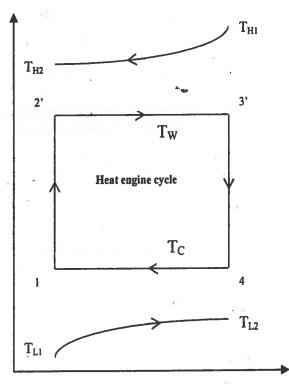


Figure 2 T-S diagram of an icreversible heat engine cycle

$$W = Q_{H} - Q_{L} W = \varepsilon_{H} C_{H} [T_{H_{1}} - T_{W}] - \varepsilon_{L} C_{L} [T_{C} - T_{L_{1}}]$$
 (6)

The power density (W_d) defined as the ratio of power produced to the maximum volume in the cycle then takes the form:

$$W_d = \frac{\varepsilon_H C_H \left[T_{H1} - T_W \right] - \varepsilon_L C_L \left[T_C - T_{L1} \right]}{V_4}$$
 (7)

The second law for an irreversible cycle requires that:

$$\oint \frac{dQ}{T} = \frac{Q_H}{T_W} - \frac{Q_L}{T_C} < 0 \tag{8}$$

One can rewrite the inequality in Eq.(8) as

$$\frac{Q_H}{T_W} - R \frac{Q_L}{T_C} = 0 \text{ with } 0 < R < 1$$
 (9)

where the internal irreversibility parameter R is defined as

$$R = \frac{S_3 - S_2}{S_4 - S_1} \tag{10}$$

By substituting equations (1) and (2) in Eq. (9), we have

$$\frac{\varepsilon_H C_H \left[T_{H1} - T_W \right]}{T_W} = R \frac{\varepsilon_L C_L \left[T_C - T_{L1} \right]}{T_C} \tag{11}$$

Assuming an ideal gas, the maximum volume in the cycle V_4 can be written as

$$V_4 = \frac{mR_g T_C}{p_{\min}} \tag{12}$$

where is the mass of the working fluid and $R_{\rm g}$ is the ideal gas constant. In the analysis, the minimum pressure ($p_{\rm min}$) in the cycle is taken to be constant[9]. The power density then becomes

$$W_d = \left(\frac{p_{\min}}{mR_g}\right) \left[\frac{\varepsilon_H C_H \left[T_{H1} - T_W\right] - \varepsilon_L C_L \left[T_C - T_{L1}\right]}{T_C}\right]$$
(13)

One can maximize the power density given in Eq.(13) with respect to T_W and T_C by using Eq.(11). The results are:

$$T_{C}^{*} = \frac{\left[\varepsilon_{H}C_{H}T_{H1}R^{2}\varepsilon_{L}C_{L}\tau + R^{2}\varepsilon_{L}^{2}C_{L}^{2}\tau^{2}T_{H1} + \tau T_{H1}\varepsilon_{H}^{2}C_{H}^{2}R + \tau^{2}T_{H1}\varepsilon_{H}C_{H}R\varepsilon_{L}C_{L} + \sqrt{\varepsilon_{H}^{2}C_{H}^{2}T_{H1}^{2}}R^{2}(R\varepsilon_{L}C_{L} + \varepsilon_{H}C_{H})\tau^{2}(\tau C_{L}\varepsilon_{L} + \varepsilon_{H}C_{H})}\right]}{\left(\varepsilon_{H}^{2}C_{H}^{2}R + \varepsilon_{H}C_{H}R^{2}\varepsilon_{L}C_{L} + 2\varepsilon_{H}C_{H}^{2}C_{L}^{2}\tau + \tau \varepsilon_{H}^{2}C_{H}^{2}\right)}$$

$$(14)$$

$$T_{W}^{\bullet} = \varepsilon_{H} C_{H} T_{H1} R^{2} \varepsilon_{L} C_{L} \tau + R^{2} \varepsilon_{L}^{2} C_{L}^{2} \tau^{2} T_{H1} + \left[\frac{\varepsilon_{H} C_{H} T_{H1} \varepsilon_{H}^{2} C_{H}^{2} R + \tau^{2} T_{H1} \varepsilon_{H} C_{H} R \varepsilon_{L} C_{L} + \left[\sqrt{\varepsilon_{H}^{2} C_{H}^{2} T_{H1}^{2} R^{2} \left(\frac{R \varepsilon_{L} C_{L}}{\varepsilon_{H} C_{H}} \right) \tau^{2} \left(\frac{\tau C_{L} \varepsilon_{L}}{\varepsilon_{H} C_{H}} \right) \right]$$

$$\varepsilon_{H}^{2} \overline{C_{H}^{2}} T_{H1} R^{2} \varepsilon_{L} C_{L} \tau + \varepsilon_{H}^{3} C_{H}^{3} \tau T_{H1} R + \left[\varepsilon_{H} C_{H} \sqrt{\varepsilon_{H}^{2} C_{H}^{2} T_{H1}^{2} R^{2} \left(\frac{R \varepsilon_{L} C_{L}}{\varepsilon_{H} C_{H}} \right) \tau^{2} \left(\frac{\tau C_{L} \varepsilon_{L}}{\varepsilon_{H} C_{H}} \right) + \left[R \varepsilon_{L} C_{L} \sqrt{\varepsilon_{H}^{2} C_{H}^{2} T_{H1}^{2} R^{2} \left(\frac{R \varepsilon_{L} C_{L}}{\varepsilon_{H} C_{H}} \right) \tau^{2} \left(\frac{\tau C_{L} \varepsilon_{L}}{\varepsilon_{H} C_{H}} \right) \right]$$

Substituting Eqs. (14) and (15) into Eq(13), the maximum power density (MPD) can be found as

(15)

$$W_{d \max} = \left(\frac{p_{\min}}{mR_g}\right) \left[\frac{\varepsilon_H C_H \left[T_{H1} - T_W^{\, \bullet}\right] - \varepsilon_L C_L \left[T_C^{\, \bullet} - T_{L1}\right]}{T_C^{\, \bullet}}\right]$$
(16)

The thermal efficiency of heat engine is given as:

$$\eta = 1 - \frac{Q_L}{Q_H} \tag{17}$$

Using Eqs. (14) and (15) into Eq(17), the efficiency at MPD (ς_{mpd}) becomes

$$\eta_{mpd} = 1 - \frac{T_C^*}{R \cdot T_W^*} \tag{18}$$

$$\eta_{mpd} = \frac{\left[\frac{C_H T_{H1} R^2 C_L \varepsilon_H \varepsilon_L \tau + C_H^2 \varepsilon_H^2 T_{H1} R^2 - \sqrt{\varepsilon_h^2 C_h^2 T_{H1}^2 R^2 \left(R \varepsilon_L C_L + \varepsilon_H C_H \right) \tau^2 \left(\tau C_L \varepsilon_L + \varepsilon_H C_H \right)} \right]}{\left(T_{H1} C_H \varepsilon_H R \left(\tau R C_L \varepsilon_L + \varepsilon_H C_H R + \varepsilon_H C_H \tau \right) \right)}$$
(19)

where $\tau = \frac{T_{L1}}{T_{H1}}$ is the cycle heat reservoir inlet

temperature ratio.

Similarly, one can maximize the power output given in Eq.(6) with respect to T_W and T_C using the condition given in Eq(11). The results are:

$$T_{wmp} = \varepsilon_{H} C_{H} T_{H1} \frac{\begin{bmatrix} \varepsilon_{H} C_{H} R \varepsilon_{L} C_{L} \tau T_{H1} + R^{2} \varepsilon_{L}^{2} C_{L}^{2} \tau T_{H1} + \\ \sqrt{T_{H1}^{2} \varepsilon_{H}^{2} C_{H}^{2} R (R \varepsilon_{L} C_{L} + \varepsilon_{H} C_{H})^{2} \tau} \end{bmatrix}}{\left(\left(\sqrt{T_{H1}^{2} \varepsilon_{H}^{2} C_{H}^{2} R (R \varepsilon_{L} C_{L} + \varepsilon_{H} C_{H})^{2} \tau} \right) \right)} \left((R \varepsilon_{L} C_{L} + \varepsilon_{H} C_{H}) \right)}$$

(20)

$$T_{cmp} = \frac{\left[\varepsilon_{H}C_{H}R\varepsilon_{L}C_{L}\tau T_{H1} + R^{2}\varepsilon_{L}^{2}C_{L}^{2}\tau T_{H1} + \left[\sqrt{T_{H1}^{2}\varepsilon_{H}^{2}C_{H}^{2}R(R\varepsilon_{L}C_{L} + \varepsilon_{H}C_{H})^{2}\tau}\right]}{(\varepsilon_{H}C_{H} + R\varepsilon_{L}C_{L})^{2}}$$
(21)

Substituting Eqs(20) and (21) into Eq(6), the maximum power can be found as

$$W_{\text{max}} = \varepsilon_H C_H [T_{H1} - T_{wmp}] - \varepsilon_L C_L [T_{cmp} - T_{L1}]$$
 (22)

The thermal efficiency at maximum power (η_{mp}) is:

$$\eta_{mp} = \frac{\left[C_{H}T_{H1}R^{2}C_{L}\varepsilon_{H}\varepsilon_{L} + C_{H}^{2}T_{H1}R\varepsilon_{H}^{2} - \int_{1}^{2} \sqrt{T_{H1}^{2}\varepsilon_{H}^{2}C_{H}^{2}R(R\varepsilon_{L}C_{L} + \varepsilon_{H}C_{H})^{2}\tau}\right]}{\left(T_{H1}C_{H}\varepsilon_{H}R(R\varepsilon_{L}C_{L} + \varepsilon_{H}C_{H})\right)}$$

(23)

3. Results and Discussion

In order to have a numerical appreciation of the results of maximum power density, detailed numerical analysis are provided and are compared with those for the maximum power objective. During the variation of any one parameter, all other parameters are assumed to be constant as given below:

$$\tau$$
=0.2, $C_H = C_L = 1$ kW/K, $\epsilon_H = \epsilon_L = 0.9$ and R=0.8.

Variations of the two efficiencies (η_{mp} and η_{mpd}) with τ are shown in Figure 3. The following comments can be summarized from the observations of this figure: (i) For the chosen values of the parameters, the thermal efficiency at MPD(η_{mod}) is always greater than the thermal efficiency at MP conditions (η_{mp}) ; (ii)The thermal efficiency advantage of the MPD conditions with respect to MP conditions ($\Delta \eta = \eta_{mpd} - \eta_{mp}$) decreases with the increase in τ (cycle heat inlet reservoir temperature Example for τ = 0.2, η_{mp} = 0.5 and η_{mpd} = 0.574, and for τ = 0.6, η_{mp} = 0.134 and η_{mpd} = 0.139. The following observations can also be seen by evaluating Eqs. (19) and (23); (iii) Both the thermal efficiencies decreases with an increase in τ and becomes zero when $\tau = R$.

Variations of the dimensionless power at MPD conditions ($P_{dmax} = W_{dmax}/(p_{min}/mR_g)\epsilon_H C_H$) and dimensionless power at MP conditions ($P_{max} = W_{max}/\epsilon_H C_H T_{H1}$) with τ are shown in Figure 4. As can be seen from the figure, P_{max} is always lower than P_{dmax} , that is if the design parameters are selected at MPD conditions instead of MP conditions, the thermal efficiency increases as much as by $\Delta \eta = \eta_{mpd} - \eta_{mp'}$, and the power increases by $\Delta P = P_{dmax} - P_{max}$. On the other hand, as τ increases, η_{mpd} and $\eta_{mp'}$, P_{dmax} and P_{max} decreases and get closer to each other and they become zero when $\tau = R$. In order to obtain positive performance in terms of thermal efficiency and power for both the MPD and MP conditions, it is necessary that τ should be less than R.

Variations of the two efficiencies (η_{mp} and η_{mpd}) with R are shown in Figure 5. From this figure, one can observe the effects of internal irreversibility on the global and optimal performances. As the internal irreversibility increases, that is , as R decreases, the global and optimal performances gradually decrease and the value of the efficiencies, η_{mp} and η_{mpd} , get very close to each other. It can also be noted that the reducing effect of the internal irreversibility on η_{mpd} is greater in comparison to η_{mp} . Thus, it is important to take precautions to reduce internal irreversibility in the design of a heat engine working at MPD conditions with respect to the one working at MP conditions.

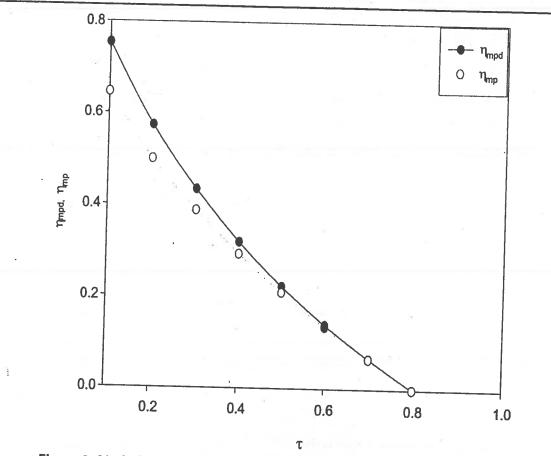


Figure 3. Variations of efficiencies of maximum power and maximum power density conditions with τ for R=0.8

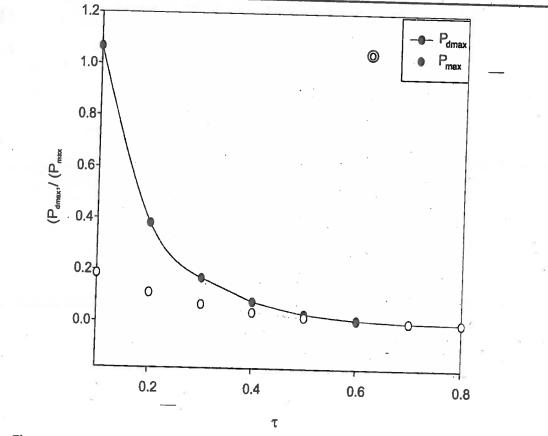
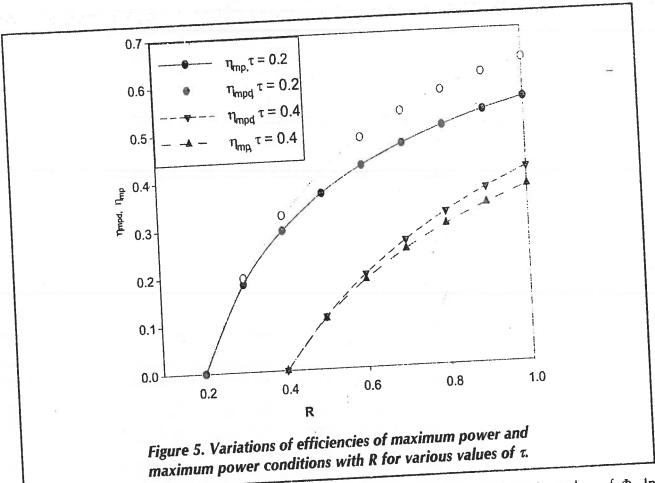


Figure 4. Variations of dimensionless maximum power density and dimensionless maximum power with τ for R = 0.8



The expressions for the efficiency, at maximum power condition and at maximum power density conditions, in terms of heat capacitance ratio can also be written as:

$$\eta_{upd} = \frac{\left[\frac{\phi R^2 \varepsilon_H \varepsilon_L \tau + \varepsilon_H^2 \phi^2 R^2 - \sqrt{\varepsilon_H^2 \phi^2 R^2 (R \varepsilon_L + \varepsilon_H \phi) \tau^2 (\tau \varepsilon_L + \varepsilon_H \phi)}}{(\phi \varepsilon_H R (\tau R \varepsilon_L + \phi \varepsilon_H R + \phi \varepsilon_H \tau))} \right] (24)$$

$$\eta_{mp} = \frac{\left[\phi R^{2} \varepsilon_{H} \varepsilon_{L} + \varepsilon_{H}^{2} \phi^{2} R - \sqrt{\varepsilon_{H}^{2} \phi^{2} R (R \varepsilon_{L} + \varepsilon_{H} \phi)^{2} \tau}\right]}{\left(\phi \varepsilon_{H} R (R \varepsilon_{L} + \phi \varepsilon_{H})\right)}$$
(25)

where $\Phi = C_H/C_L$, is the heat capacitance ratio.

The variation of efficiencies with heat capacitance ratios(Φ) is shown in Figure 6.

It can be seen from the figure that as the value of Φ increases, $\eta_{\mbox{\tiny mpd}}$ increases. It is noticeable that the rate of increase in $\eta_{\mbox{\tiny mpd}}$ is more till the value of $\Phi \leq 0.4$ and there after the rate of increment slows down and becomes almost constant when becomes \geq 1. For example η_{mpd} = 0.578, when Φ = 0.1, τ = 0.2 : η_{mpd} = 0.635,

when $\Phi=0.9,\, \tau=0.2$ and $\eta_{mpd}=0.64$ when $\Phi=1.4$ and $\tau=0.1.$ It can also be observed that

 η_{mp} remains constant with the value of $\Phi.$ In order to obtain positive performance output for both maximum power density and maximum power conditions, the value of Φ should approximately be unity. Such case can be seen by evaluating equations (24) & (25).

The size of a heat engine can be characterized by the maximum volume in the cycle, ie. V_4 . For turbo machinery, Sahin et al[12] discussed that the maximum volume in the cycle characterizes the engine size.

The ratio of the maximum volume at MPD to the one at MP can be written as:-

$$\frac{\left(V_{\text{max}}\right)_{\text{mpd}}}{\left(V_{\text{max}}\right)_{\text{mpd}}} = \frac{\left[\left(\varepsilon_{H}C_{H}R^{2}\varepsilon_{L}C_{L}\tau + R^{2}\varepsilon_{L}^{2}C_{L}^{2}\tau^{2}\right) + \tau\varepsilon_{H}^{2}C_{H}^{2}R + \tau^{2}\varepsilon_{H}C_{H}R\varepsilon_{L}C_{L} + \left(\sqrt{\varepsilon_{H}^{2}C_{H}^{2}R^{2}(R\varepsilon_{L}C_{L} + \varepsilon_{H}C_{H})}\tau^{2}(\tau C_{L}\varepsilon_{L} + \varepsilon_{H}C_{H})\right)\right]}{\left(\left(\varepsilon_{H}^{2}C_{H}^{2}R + \varepsilon_{H}C_{H}R^{2}\varepsilon_{L}C_{L} + 2\varepsilon_{H}C_{H}R\varepsilon_{L}C_{L}\tau + \frac{1}{2}C_{H}^{2}C_{L}^{2}\tau + \tau\varepsilon_{H}^{2}C_{H}^{2}C_{L}^{2}\right)}\right]} \\
= \frac{\left(V_{\text{max}}\right)_{\text{mpd}}}{\left(V_{\text{max}}\right)_{\text{mpd}}} = \frac{\left[\left(\varepsilon_{H}C_{H}R\varepsilon_{L}C_{L}\tau + R^{2}\varepsilon_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{H}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{H}^{2}C_{L}^{2}\tau + \varepsilon_{H}^{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{H}^{2}C_{L}^{2}\tau + \varepsilon_{H}^{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{H}^{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{L}^{2}C_{L}^{2}\tau + \frac{1}{2}C_{L}^{2}C$$

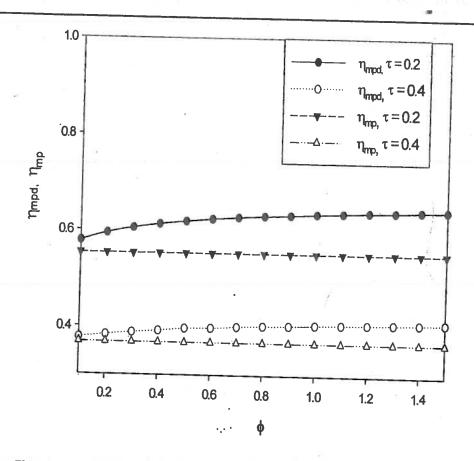


Figure 6. Variations of efficiencies of maximum power and maximum power density conditions w ith heat capacitance ratio for various values of τ .

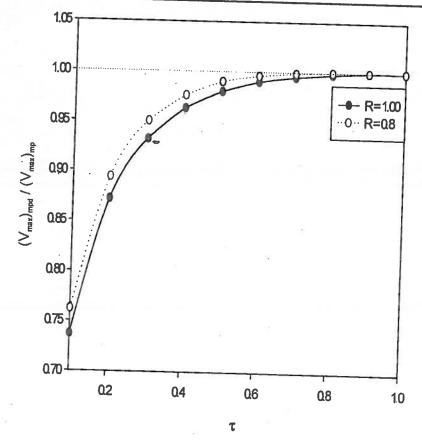


Figure 7. Variations of volume ratio with τ for different values of R.

Variations of $(V_{max})_{mpd}$ / $(V_{max})_{mp}$ are plotted with respect to τ in Figure 7. As can be seen, the ratio of maximum volumes in the cycle is always less than unity. As a result, at MPD conditions engine sizes will always be smaller than the one operating at MP. Moreover as the internal irreversibility increases, that is, as R decreases, the engine size advantage of the MPD conditions decreases.

4. Conclusions

The performance of an irreversible Carnot Heat engine cycle coupled to variable temperature heat reservoirs with heat transfer irreversibility in the hot and cold side heat exchangers was analyzed by taking the power density as the optimization objective.

Comparisons between maximum power density conditions and maximum power conditions were made. The effects of the cycle irreversibility parameter on the efficiency and power output at MP and MPD conditions are studied by detailed numerical examples. The analysis showed that the thermal efficiency at MPD conditions is greater than the one at MP conditions. However, the thermal efficiency advantage of the MPD conditions with respect to MP condition decreases as the internal irreversibility increases. The results indicate that the reducing effect of the internal irreversibility on the thermal efficiency at MPD conditions is greater in comparison to the one working at MP conditions. Further more, it is shown that engine sizes designed at MPD conditions would be smaller than those operating at MP conditions. The results presented in this analysis generalize the results of the previous studies and may provide guidelines for determination of the optimal design and operating conditions of real heat engines.

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References:

- 1. Curzon, F.L.; Ahlborn, B. "Efficiency of a Carnot Engine at maximum power output". American Journal of Physics, Vol. 43, 1975, pp. 22-24.
- 2. Wu, C. "Power optimization of an endoreversible Brayton gas turbine heat engine". Energy Convers. Mgmt, Vol. 31, 1991, pp.561-565.
- 3. Bezan, A. "Entropy generation minimization: The New Thermodynamics of Finite-Size Devices and Finite Time Processes". J. Applied Physics, Vol.79, 1996, pp.1191-1218.
- 4. Wu, C.; Chen, L.; Sun, F. "Performance of a regenerative Brayton heat engine". Energy Int. Journal, Vol 21, 1996, pp.71-76.
- 5. Chen, L; Sun, F; Wu, C. "Performance analysis of an irreversible Brayton heat engine". J. Inst. Energy, Vol70, 1997, pp.2-8.
- Chen, L; Sun, F; Wu, C; Kiang, R.L. "Theoretical analysis of the performance of a regenerative closed Brayton cycle with internal irreversibilities". Energy Conv & Mgmt, Vol. 18, 1997, pp.871-877.
- 7. Cheng, C.Y.; Chen, C.K. "Power optimization of an irreversible Brayton heat engine". Energy Sources; Vol. 19, 1997, pp.461-474.
- 8. Kaushik, S.C.; Kumar, P. "Finite time optimization of an irreversible Rankine cycle heat pump system". Indian Journal of Pure & Applied Physics, Vol. 39, 2001, pp.628-635.
- 9. Chen, J. "The maximum power output and maximum efficiency of an irreversible Carnot heat engine". J Phy D: App Physics, Vol. 27, 1994, pp.1144-1149.
- 10. Chen; L.; Ni, N.; Cheng, G.; Sun, F.; Wu, C. "Performance analysis for a real closed regenerated Brayton cycle via methods of finite time thermodynamics". Int. J. Ambient Energy; Vol. 20, 1999, pp. 95-104.

- 11. Chen, L.; Sun, F.; Wu, C. "Effect of heat resistance on the performance of closed gas turbine regenerative cycles". Int. J. Power and Energy Systems, Vol. 19, 1999, pp.141-145.
- 12. Sahin, B.; Kodal, A.; Yavuz, H. "Efficiency of a Joule Brayton engine at maximum power density". J Phy D: App Physics, Vol. 28, 1995, pp.1309-1312.
- 13. Sahin, B.; Kodal, A.; Yavuz, H. "Maximum power density analysis of an irreversible Joule Brayton engine". J Phy D: App Physics, Vol. 29, 1996, pp.1162-1171.
- 14. Maheshwari, G.; Khandwawala, A. I.; Kaushik, S. C. "Maximum power density analysis for an irreversible radiative heat engine". The Int. J. Ambient Energy, Vol.26, 2005, pp.71-80.
- 15. Erbay, L.B.; Sisman, A.; Yavuz, H. "Analysis of Ericsson cycle at maximum power density". ECOS'96 Stockholm, 1996, pp.175-178.
- Chen, L.; Lin, J.; Sun, F. "Efficiency of an Atkinson engine at maximum power density". *Energy Convers Mgmt.*, Vol. 9, 1998, pp.337-341.
- 17. Sahin, B.; Kodal, A. "Maximum power density analysis for irreversible combined Carnot cycles". J Phy D: Appl Phys., Vol. 22, 1999, pp.2958-2963.
- 18. Chen, L,; Zheng, J.; Sun, F.; Wu, C. "Performance comparison of an endoreversible closed variable temperature heat reservoir Brayton cycle under maximum power density and maximum power conditions". Energy Convers Mgmt., Vol. 42, 2001, pp.337-341.
- 19. Maheshwari, G.; Khandwawala, A. I.; Kaushik, S. C. "A comparative performance analysis of an endoreversible heat engine with thermal reservoir of finite heat capacitance under maximum power density and maximum power conditions". The Int. J. Ambient Energy, Vol. 26, 2005, pp.147-154.

^{*}Department of Mechanical Engineering, Institute of Engineering & Technology, Devi Ahilya University, Khandwa Road, Indore-452017, INDIA. Tel. (0731)2361116-7, 2764385 • govind_maheshwari2001@yahoo.com