OPTIMIZATION OF SUSPENSION SYSTEM TO MINIMIZE THE SHOCK RESPONSE - A THEORETICAL ASPECT

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ABSTRACT

The shock response is important criterion to be considered in vehicle design for passenger comfort as well as the goods carried by it. Most of the work is carried out for finding the shock response spectrum for a single degree of freedom system. This paper deals with the reduction of shock response of an equipment mounted on a vehicle. A procedure of the calculation of the shock response of the equipment mounted in the vehicle is presented and the optimization procedure is given. The procedure can be used for the mounting of the equipment in the vehicle. Depending on the shape and duration of the shock pulse, optimization can be carried out. Shock response of a linear 4-DOF vehicle model is minimized by optimizing the equipment suspension system parameters for the excitation due to ground undulations. The vehicle is modeled as a system of linear springs and the goal is to minimize the motion of the equipment being carried in the vehicle. The kinetic energy of equipment is chosen as an objective function. The stiffness and positions of the equipment suspension spring are the design variables and suspension system of the vehicle is considered as the design parameter. Response of the mounted equipment is calculated as a transient response to half sine wave through the use of Duhamel's integral for each of the modes of vibrations. The effect of the variation of each design parameter is considered separately and the optimum value of each variable is fixed, a program in MATLAB is developed.

Key Words: Shock response, optimization, stiffness, kinetic energy.

Nomenclature: CG_1, CG_2 = Centre of gravity of body 1 & body 2, M_1 and M_2 = Mass of body 1 and 2 (Kg)

K_1, K_2 = Spring Stiffness between body 1 and ground (N/m).

K_3, K_4 =Spring Stiffness between body 2 and body 1 (N/m)

a,b,c,d-Distances of springs K_1, K_2, K_3, K_4 from CG_1(m)

e,f - Distances of springs K_1 and K_2 from CG_2 (m)

I_1, I_2 = Mass moment of inertia of body 1 and body 2 (Kg-m^2)

X_1, X_3 = Bouncing motion of mass M_1 and M_2 respectively (m)

X_2, X_4 = Pitching motion of mass M_1 and M_2 respectively (rad)

$@_i$ = Frequency of the i^{th} mode of vibration (rad/sec)

$\{U\}_i$ = Eigen vector for the i^{th} mode of vibration, V=Vehicle speed (Km/hr)

T= Shock pulse duration (sec), @=Frequency of half sine wave= π/T (rad / sec)

t_1= Time delay between front and rear wheels (sec)

$Y_1(t)$, $Y_2(t)$= Shock Inputs for front and rear wheels

$beta_1 = K_3/K_4$, $beta_2 = c/d$, $beta_3 = e/f$

l = Wheel base (m)

$K.E.$= Kinetic Energy ($_{N\text{-}m}$)

1. INTRODUCTION

At large sensitive equipments are transported through vehicles which may run over bumps (ground undulations). Similarly the ships, buildings, space crafts and launched vehicles are also subjected to under water explosions, earth quakes, and pyroshocks respectively. To circumvent the potential damage of the equipments shock response should be minimized. Estimation of the damage potential of a shock pulse was calculated by shock response spectrum in 1930's by Biot, Irvine [1] for an array of single degree-of-freedom (SDOF) system. The response was found in the time domain using convolution integral. The end result was represented using natural frequency domain for half-sine base input. Alexander [2] employed series of SDOF linear oscillators with different frequencies and for multi degree freedom system superposition technique was used. Underwater explosion and surface vessels response was studied by Reid [3] for varies factor such as accelerations and displacements leading to stresses in a system and subsequent modifications in the pilot design. Cunniff et al. [4] reported design values for structure subjected to transient motion excitation.

Most of the research work reported shock response spectrum analysis for a SDOF system. The accuracy of the results may be enhanced by incorporating more number of degree-of-freedoms (DOF) for a system.

This paper deals with a system (4 –DOF) subjected to shock input (ground undulations) aiming at minimization of vehicle mounted equipment response.

2. MATHEMATICAL MODEL

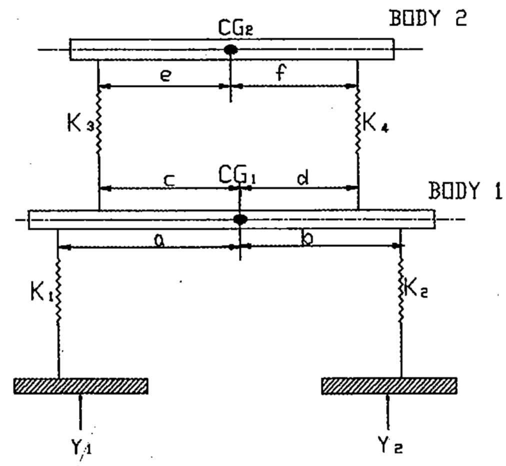


Fig.1 Vehicle model (4-DOF) (Equilibrium condition)

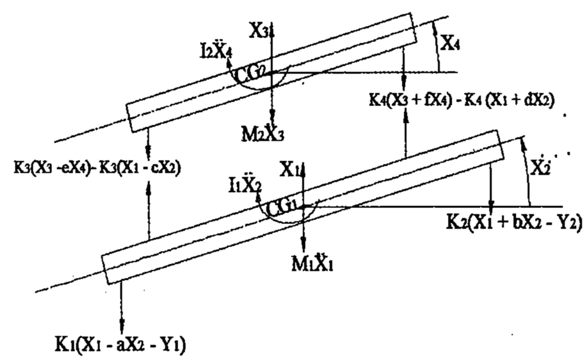


Fig.2 Free body diagram (Dynamic condition)

2.1 Description of Model

A linear vehicle model (4-DOF) at equilibrium and dynamic conditions is shown in Fig.1 and Fig.2 respectively. The body 2 which is equipment is mounted on body 1 (vehicle body). Rigid bodies of masses M_1 and M_2 representing vehicle body and equipment can perform translatory (bouncing motion) and rotary motion (pitching motion) described by X_1, X_3 and X_2, X_4 respectively. Passive springs of suspension are modeled as a linear spring element K_1, K_2, K_3, K_4. The vehicle is moving horizontally with constant velocity 'V'. Shock inputs $Y_1(t)$, $Y_2(t)$ in the form of half sine wave having 'A' as amplitude and duration 'T' are given to the wheels and response of body 2 to this input is found. Equations of motion in the matrix form are given by Eq. (1).

$$\begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 \\ 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & I_2 \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \\ \ddot{X}_4 \end{Bmatrix} +$$

$$\begin{bmatrix} (K_1+K_2+K_3+K_4) & (-K_1a+K_2b-K_3c+K_4d) & -(K_3+K_4) & (K_3e-K_4f) \\ (-K_1a+K_2b-K_3c+K_4d) & (K_1a^2+K_2b^2+K_3c^2+K_4d^2) & (K_3c-K_4d) & -(K_3ce+K_4df) \\ -(K_3+K_4) & (K_3c-K_4d) & (K_3+K_4) & (-K_3e+K_4f) \\ (K_3e-K_4f) & -(K_3ce+K_4df) & (-K_3e+K_4f) & (K_3e^2+K_4f^2) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix}$$

$$\begin{bmatrix} K_1Y_1+K_2Y_2 \\ -K_1Y_1a+K_2Y_2b \\ 0 \\ 0 \end{bmatrix} \qquad \text{.................. (1)}$$

Considering unit impulse at each wheel, initial velocities are computed considering linear and angular momentum. Principle of impulse and momentum for the plane motion of a rigid body is used. When the vehicle is at rest, initial linear and angular momentums are zero. Under this situation, the unit impulse is applied at front and rear wheels one by one. The two cases are

Case i) - When unit impulse is applied at front wheels (Fig.3) of body with mass M_1, the initial linear velocity $x_1^{\cdot}$ is obtained as $1/M_1$ and angular velocity $x_2^{\cdot}$ is obtained as $a/(I_1)$

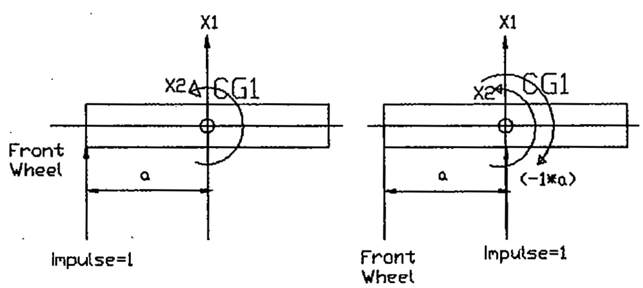
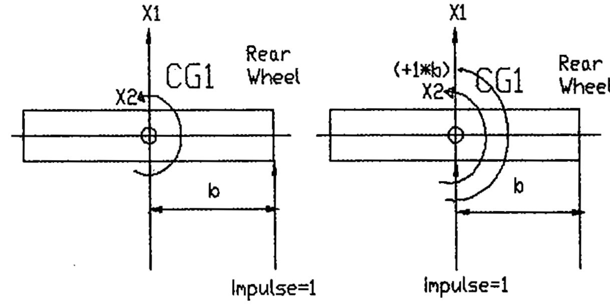


Fig.3 Unit impulse applied (a) at front wheels (body mass $1M$) (b) Equivalent System

Case ii) - When unit impulse is applied at rear wheels (Fig.4) of body with mass M_1, the initial linear velocity $x_1^{\cdot}$ is obtained as $1/M_1$ and angular velocity $x_2^{\cdot}$ is obtained as $b/(I_1)$.

Substituting these values of initial velocities, the response of system to unit impulse given at front and rear wheels of body with mass M_1 is given by Eq. (2) and Eq. (3) respectively.



(a) ((b)

Fig.4 Unit impulse applied (a) at rear wheels (body mass M_1) (b) Equivalent System

$$\begin{Bmatrix} h_1(\tau) \\ h_2(\tau) \\ h_3(\tau) \\ h_4(\tau) \end{Bmatrix} = \frac{1}{\omega_1}\sin\omega_1 t\,(U_{11}-aU_{21}) \times \begin{Bmatrix} U_{11} \\ U_{21} \\ U_{31} \\ U_{41} \end{Bmatrix} + \frac{1}{\omega_2}\sin\omega_2 t\,(U_{12}-aU_{22}) \times \begin{Bmatrix} U_{12} \\ U_{22} \\ U_{32} \\ U_{42} \end{Bmatrix} +$$

$$\frac{1}{\omega_3}\sin\omega_3 t\,(U_{13}-aU_{23}) \times \begin{Bmatrix} U_{13} \\ U_{23} \\ U_{33} \\ U_{43} \end{Bmatrix} + \frac{1}{\omega_4}\sin\omega_4 t\,(U_{14}-aU_{24}) \times \begin{Bmatrix} U_{14} \\ U_{24} \\ U_{34} \\ U_{44} \end{Bmatrix}$$

$$\begin{Bmatrix} h_1(\tau) \\ h_2(\tau) \\ h_3(\tau) \\ h_4(\tau) \end{Bmatrix} = \frac{1}{\omega_1}\sin\omega_1 t\,(U_{11}+bU_{21}) \times \begin{Bmatrix} U_{11} \\ U_{21} \\ U_{31} \\ U_{41} \end{Bmatrix} + \frac{1}{\omega_2}\sin\omega_2 t\,(U_{12}+bU_{22}) \times \begin{Bmatrix} U_{12} \\ U_{22} \\ U_{32} \\ U_{42} \end{Bmatrix} +$$

$$\frac{1}{\omega_3}\sin\omega_3 t\,(U_{13}+bU_{23}) \times \begin{Bmatrix} U_{13} \\ U_{23} \\ U_{33} \\ U_{43} \end{Bmatrix} + \frac{1}{\omega_4}\sin\omega_4 t\,(U_{14}+bU_{24}) \times \begin{Bmatrix} U_{14} \\ U_{24} \\ U_{34} \\ U_{44} \end{Bmatrix} \qquad \text{.......................... (3)}$$

Using Duhamel's Integral (Eq.4), the response of 4-DOF system to shock input is computed.

$$g(t) = \int_{-\infty}^{\infty} F(\tau)h(t-\tau)d\tau \qquad \text{.......................... (4)}$$

Where,

$F(\tau)$ (is arbitrary input function with varying amplitude at time 't' and $h(t)$ is characteristic impulse response of a system.

In case of arbitrary excitation the response of a system is purely transient. By the principle of superposition, the output signal at time 't' consists of the sum (or superposition) of all the suitably scaled and time delayed impulse responses.

3. SHOCK RESPONSE COMPUTATION

Shock Response computation for ground undulations (i.e. front and rear wheels) $Y_1(t)$ and $Y_2(t)$ respectively with half pulse of amplitude "A" and duration "T" is explained below. The excitation functions at two wheels are delayed by time equal to wheel base divided by velocity of vehicle. The excitation functions for the front and rear wheels are $Y_1(t)$ and $Y_2(t)$ as given by Eq. (5).

Let,

$$\left.\begin{array}{ll} Y_1(t) = A\sin(\omega t) & \text{for } t \le T \\ Y_1(t) = 0 & \text{for } t > T \end{array}\right\} \quad \text{.........................(5)}$$

$$Y_2(t) = Y_1(t - t_1) = A\sin[\omega(t - t_1)]$$

$$= A\sin[\omega(t - l/V)] \text{ , where } t_1 = l/V$$

Where A and t are amplitude of the bump and the time interval.

The characteristic impulse response for the present 4-DOF model is derived in Eq.(2) and Eq.(3).

The impulse at front wheels of mass M_1 is $F_1(\tau)$

$$F_1(\tau) = K_1 Y_1(t) \quad \text{.................................(6)}$$

The impulse at rear wheels of mass M_1 is $F_2(\tau)$

$$F_2(\tau) = K_2 Y_2(t) \quad \text{.................................(7)}$$

Using Eq. (6) and Eq. (7) and impulse response Eq. (2) and Eq. (3), shock response is computed during the pulse time ($0 \le t \le T$) and subsequent to termination of pulse ($t > T$) Eq. (8) and Eq. (9).

For $0 \le t \le T$

$$\begin{aligned} X_1(t) = (K_1 \times A) \times \{ & 1/\omega_1 \times (U_{11} - aU_{21}) \times U_{11} \times [[(\omega)\sin(\omega_1 t) - \omega_1 \sin(\omega t)]/(\omega^2 - \omega_1^2)] \\ & + 1/\omega_2 \times (U_{12} - aU_{22}) \times U_{12} \times [[(\omega)\sin(\omega_2 t) - \omega_2 \sin(\omega t)]/(\omega^2 - \omega_2^2)] \\ & + 1/\omega_3 \times (U_{13} - aU_{23}) \times U_{13} \times [[(\omega)\sin(\omega_3 t) - \omega_3 \sin(\omega t)]/(\omega^2 - \omega_3^2)] \\ & + 1/\omega_4 \times (U_{14} - aU_{24}) \times U_{14} \times [[(\omega)\sin(\omega_4 t) - \omega_4 \sin(\omega t)]/(\omega^2 - \omega_4^2)]\} \\ + (K_2 \times A) \times \{ & 1/\omega_1 \times (U_{11} + bU_{21}) \times U_{11} \times [[(\omega)\sin(\omega_1 t)\cos(\omega t_1) - \omega_1 \sin(\omega t - \omega t_1) - \omega_1 \cos(\omega_1 t)\sin(\omega t_1)]/(\omega^2 - \omega_1^2)] \\ & + 1/\omega_2 \times (U_{12} + bU_{22}) \times U_{12} \times [[(\omega)\sin(\omega_2 t)\cos(\omega t_1) - \omega_2 \sin(\omega t - \omega t_1) - \omega_2 \cos(\omega_2 t)\sin(\omega t_1)]/(\omega^2 - \omega_2^2)] \\ & + 1/\omega_3 \times (U_{13} + bU_{23}) \times U_{13} \times [[(\omega)\sin(\omega_3 t)\cos(\omega t_1) - \omega_3 \sin(\omega t - \omega t_1) - \omega_3 \cos(\omega_3 t)\sin(\omega t_1)]/(\omega^2 - \omega_3^2)] \\ & + 1/\omega_4 \times (U_{14} + bU_{24}) \times U_{14} \times [[(\omega)\sin(\omega_4 t)\cos(\omega t_1) - \omega_4 \sin(\omega t - \omega t_1) - \omega_4 \cos(\omega_4 t)\sin(\omega t_1)]/(\omega^2 - \omega_4^2)]\} \end{aligned}$$

Similarly $X_2(t), X_3(t), X_4(t)$ are found out for $0 \le t \le T$. ....(8)

For $t > T$

$$\begin{aligned} \{X_1(t)\} = \left(\frac{K_1 \times A}{2}\right) \times \{ & 1/\omega_1 \times (U_{11} - aU_{21}) \times U_{11} \times (\omega - \omega_1)[\sin[(\omega + \omega_1)T - \omega_1 t]] - (\omega + \omega_1)[\sin[(\omega - \omega_1)T + \omega_1 t]] + 2\omega\sin(\omega_1 t)/\omega^2 - \omega_1^2 \\ & + 1/\omega_2 \times (U_{12} - aU_{22}) \times U_{12} \times (\omega - \omega_2)[\sin[(\omega + \omega_2)T - \omega_2 t]] - (\omega + \omega_2)[\sin[(\omega - \omega_2)T + \omega_2 t]] + 2\omega\sin(\omega_2 t)/\omega^2 - \omega_2^2 \\ & + 1/\omega_3 \times (U_{13} - aU_{23}) \times U_{13} \times (\omega - \omega_3)[\sin[(\omega + \omega_3)T - \omega_3 t]] - (\omega + \omega_3)[\sin[(\omega - \omega_3)T + \omega_3 t]] + 2\omega\sin(\omega_3 t)/\omega^2 - \omega_3^2 \\ & + 1/\omega_4 \times (U_{14} - aU_{24}) \times U_{14} \times (\omega - \omega_4)[\sin[(\omega + \omega_4)T - \omega_4 t]] - (\omega + \omega_4)[\sin[(\omega - \omega_4)T + \omega_4 t]] + 2\omega\sin(\omega_4 t)/\omega^2 - \omega_4^2\} \\ + \left(\frac{K_2 \times A}{2}\right) \times \{ & 1/\omega_1 \times (U_{11} + bU_{21}) \times U_{11} \times (\omega - \omega_1)[\sin[\omega(T - t_1) + \omega_1(T - t)] + \sin(\omega_1 t + \omega t_1)] - (\omega + \omega_1)[\sin[\omega(T - t_1) - \omega_1(T - t)] - \sin(\omega_1 t - \omega t_1)]/\omega^2 - \omega_1^2 \\ & + 1/\omega_2 \times (U_{12} + bU_{22}) \times U_{12} \times (\omega - \omega_2)[\sin[\omega(T - t_1) + \omega_2(T - t)] + \sin(\omega_2 t + \omega t_1)] - (\omega + \omega_2)[\sin[\omega(T - t_1) - \omega_2(T - t)] - \sin(\omega_2 t - \omega t_1)]/\omega^2 - \omega_2^2 \\ & + 1/\omega_3 \times (U_{13} + bU_{23}) \times U_{13} \times (\omega - \omega_3)[\sin[\omega(T - t_1) + \omega_3(T - t)] + \sin(\omega_3 t + \omega t_1)] - (\omega + \omega_3)[\sin[\omega(T - t_1) - \omega_3(T - t)] - \sin(\omega_3 t - \omega t_1)]/\omega^2 - \omega_3^2 \\ & + 1/\omega_4 \times (U_{14} + bU_{24}) \times U_{14} \times (\omega - \omega_4)[\sin[\omega(T - t_1) + \omega_4(T - t)] + \sin(\omega_4 t + \omega t_1)] - (\omega + \omega_4)[\sin[\omega(T - t_1) - \omega_4(T - t)] - \sin(\omega_4 t - \omega t_1)]/\omega^2 - \omega_4^2\} \end{aligned}$$

Similarly $X_2(t)$, $X_3(t)$, $X_4(t)$ are computed for $t > T$. ....(9)

4. VEHICLE SUSPENSION OPTIMIZATION

For optimization of K_3, K_4, c, d, e, and f, the standard vehicle data is assumed (Table 1).

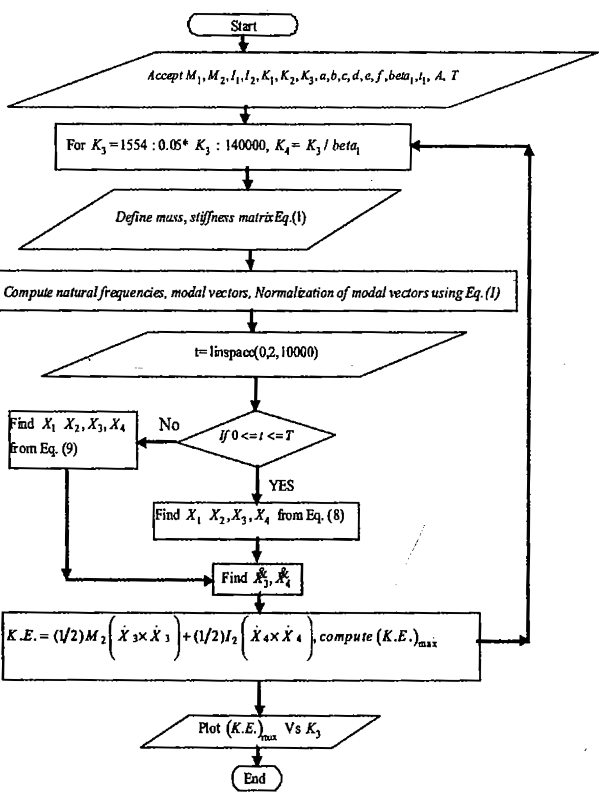
Table 1: Vehicle Parameters

A=5cm	M_1=5000Kg	M_2=1000Kg	I_1=22,500 Kg-m^2	I_2=1,092Kg-m^2
$beta_1 = K_3/K_4$ =0.5385	a=2.5m	b=1.5m	c=1m	d=1m
T=0.37715sec	e=1.3m	f=0.7m	l=4m	V=30Km/hr
$t_1 = l/V$=0.48sec	K_1=296081N/m	K_2=493468N/m	K_3=31089N/m	K_4=57737N/m

For optimization, the kinetic energy ($K.E.$) of the equipment (body 2) given by Eq. (10) is taken as objective function which is to be minimized. MATLAB code is used, the flow chart given below for plotting K_3 Vs.$K.E.$,the same flow chart is used to vary $beta_1$, $beta_2$ and $beta_3$ to calculate the kinetic energy.

$$K.E. = 1/2 \times \left[(M_2 \times \dot{X}_3^2) + (I_2 \times \dot{X}_4^2) \right] \quad \text{..........................(10)}$$

Flow chart - computing K_3 V_s K.E.



Result and Discussion

Figures 5 to 8 show the effect of variations of various parameters on the kinetic energy of the equipment.

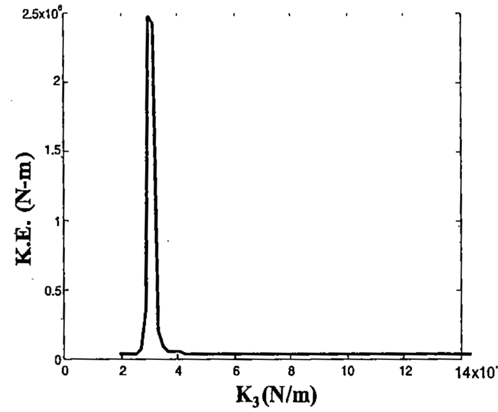


Fig.5 Variation of objective function (Kinetic Energy) with Stiffness K_3

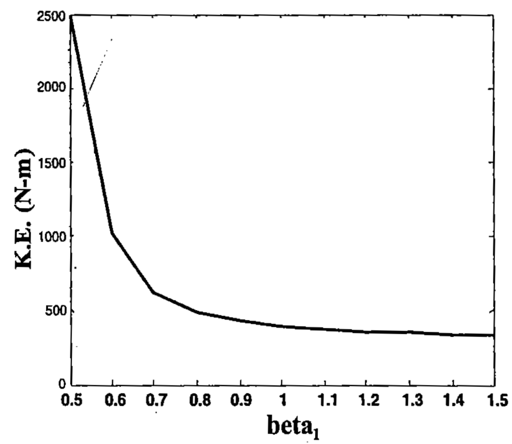


Fig.6 Variation of objective function (Kinetic Energy) with Stiffness K3

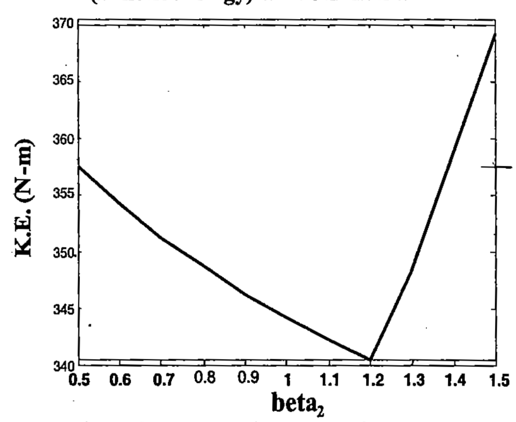


Fig.7 Variation of objective function (Kinetic Energy) with beta2

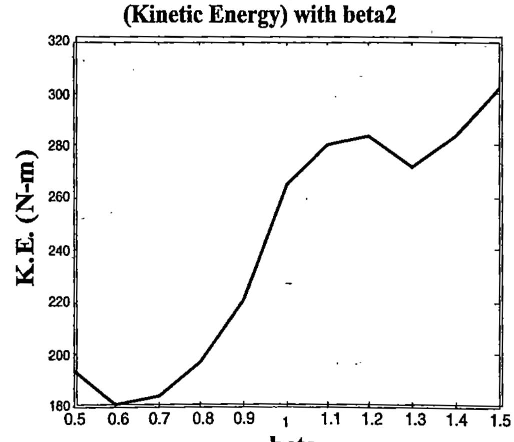


Fig.8 Variation of objective function (Kinetic Energy) with beta3

EFFECT OF VARIATION OF K_3

Initially K_3 is varied along with K_4 keeping all other parameters constant. Also the ratio $beta_1$= K_3/K_4 is kept constant. As shown in Fig.5, it is seen that kinetic energy is very high for particular band of values of K_3. This band values should be avoided and physically realizable value for K_3 *to* be chosen. The chosen value of K_3 is 15545N/m and used for further analysis. With this value of K_3, kinetic energy works out to be 1623.9*N-m*.

EFFECT OF VARIATION OF $BETA_1$

When $beta_1$ is varied as shown in Fig.6 it is seen that kinetic energy significantly reduces for values of $beta_1$ upto1.2. Therefore the value of $beta_1$ should be greater than this. The value of $beta_1$ chosen is 1.5. With this value of $beta_1$, stiffness K_4 is fixed and used further. With these values of K_3, K_4 and $beta_1$, kinetic energy works out to be 344.11*N-m*.

EFFECT OF VARIATION OF $BETA_2$

The sum of distances *c* and *d* is fixed from the space consideration and ratio $beta_2$ *c/d*= is varied. The value of $beta_2$ will determine the values of distances *c* and *d*. The effect of variation of is shown in Fig.7, the minimum value of kinetic energy occurs at a particular value of $beta_2$ which is 1.2. This value of $beta_2$ is used to calculate distances *c* and *d*. With these values of K_3, K_4, C and *d*, kinetic energy works out to be 340.47*N-m*.

EFFECT OF VARIATION OF $BETA_3$

The sum of distances *e* and *f* is fixed from the space consideration and ratio $beta_3$ *ef*=is varied. The value of $beta_3$ will determine the values of distances *e* and *f*. The effect of variation of $beta_3$ is shown in Fig.8,. It is seen that kinetic energy reduces for the value of $beta_3$ up to 0.6 and is minimum at 0.6 and then increases continuously. Hence the value of $beta_3$ is chosen as 0.6. This value of $beta_3$ is used to calculate distances e and *f*. With these values of K_3, K_4, *c, d, f*, kinetic energy works out to be 180.47. N-m.

Conclusion

Theoretical aspect of the calculation of the shock response of the equipment mounted in the vehicle is presented and optimized in the paper. Depending on the shape and duration of the shock pulse, optimization can be carried out effectively.

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