# Dispersion Analysis of Plume from Stacks

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*Abstract*

*Global realization of the need to control the ever-increasing nuisance from pollution has forced the development of analytical tools to predict GLC (Ground Level Concentration). While there has been considerable progress in this direction, none of the present-day analytical techniques, however sophisticated, can accurately represent meteorological conditions, nature of terrain etc. hence the accuracy of predicted GLC varies widely. In many simple cases, present-day analytical techniques yield acceptable results as confirmed from field observations. Atmospheric dispersion models are mathematical expressions which describe atmospheric processes in order to relate emission rate to atmospheric concentration of pollutants. In the present paper, Gaussian Plume Model (GPM) and its variations are used for ambient air quality predictions. Relation between pollutant concentration at a point and rate of emission of pollutant discharge from stack is established. Maximum GLC along plume axis is estimated. Plume rise is estimated which is further used for calculation of effective stack height.*

**Keywords:** GLC, Plume, Atmospheric turbulence, Effective stack height

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## 1. Introduction

Physical height of a stack is designed to demonstrate proper dispersion of flue gases so as to attain satisfactory ambient air quality. Attainment of satisfactory air quality is predicted through dispersion modeling. Considering the scarcity of data in Indian conditions, GPM is widely used for air quality modeling as compared to other models. The application of GPM requires knowledge of several parameters, namely emission release rate, atmospheric turbulence, wind speed, dispersion coefficients, effective stack height and mixing height etc. The experience so far, has shown that values of these parameters are often adopted from other countries without understanding their applicability in Indian context. It has also been observed that various forms of GPM are used without providing any reasonable justification in doing so [1]. In India, 'National Ambient Air Quality Standards' have been set by Central Pollution Control Board (CPCB). This paper on dispersion modeling is an effort to streamline the modeling procedure in Indian context.

### 1.1 Gaussian Plume Model

Consider a single chimney emitting pollutants at a constant rate as shown in

Fig1. Based on broad correlation with experimental observations, both horizontal and vertical spreads of plume are assumed to follow Gaussian curve [2, 3 and 4]. With this assumption expression of pollutant concentration is:

$$\gamma = C \cdot \exp[-y^2/2\sigma^2{}_y] \cdot \exp[-z^2/2\sigma^2 z] \qquad (1)$$

Where, C is a constant and y, z are distances along directions perpendicular to wind axis as shown in Fig 1. $\sigma_y$ and $\sigma_z$ are standard deviations of plume concentration in y and z directions respectively; at the location where GLC is required. The area under this curve for $\gamma$ is given by:

$$C\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\exp[-y^2/2\sigma_y^2\right]\exp[-z^2/2\sigma_z^2]\, dydz \qquad (2)$$

The magnitude of this area signifies pollutant concentration at a point:

$$\gamma = \frac{Q_P}{2\pi\sigma_y\sigma_z\overline{U}}\exp\left[-\frac{y^2}{2\sigma_y^2}\right].\exp\left[-\frac{z^2}{2\sigma_z^2}\right] \qquad (3)$$

$Q_P$ is the rate of emission of pollutant discharge in gm/s and $\gamma$ is pollutant concentration in gm/m$^3$.

Consider a typical plume spread as shown in Fig 2. It is assumed that when gaseous pollutants strike the ground at point C, they are totally reflected although some quantity would adhere to ground surface or vegetation. Hence at location E at a height z above ground, there is a portion of pollutant reaching directly and an additional quantity reflected off ground. The latter is taken as equal to the direct component calculated at the same distance from source but at a distance z below ground, i.e. at a distance ($H_e + z$) from plume axis. Physical height of a chimney together with plume rise ($\Delta H$) is termed its effective height ($H_e$) which is the height used in plume dispersion expressions. Hence expression for pollutant concentration becomes:

$$\gamma(x,y,z;H_e)=\frac{Q_p}{2\pi\sigma_y\sigma_z\overline{U}}\exp[-y^2/2\sigma_y^2]\left\{\exp\left[-\frac{1}{2}\left(\frac{z+H_e}{\sigma_z}\right)^2\right]+\exp\left[-\frac{1}{2}\left(\frac{z-H_e}{\sigma_z}\right)^2\right]\right\} \tag{4}$$

For maximum GLC along plume axis, putting y = z = o in expression (4):

$$\gamma(x,0,0;H_e)=\frac{Q_p}{\pi\sigma_y\sigma_z\overline{U}}\exp\left[-\frac{1}{2}\left(\frac{H_e}{\sigma_z}\right)^2\right] \tag{5}$$

If $\sigma_y$ and $\sigma_z$ are expressed as power law functions:

$$\sigma_y = A.x^p \text{ and } \sigma_z = E.x^p \tag{6}$$

Where, A, E and $p$ are constants. Putting values of $\sigma_y$ and $\sigma_z$ in expression (5):

$$\gamma(x,0,0;H_e)=\frac{Q_p}{\pi AE\overline{U}}x^{-2p}\exp\left[-\frac{1}{2}\left(H_e^2/E^2x^{2p}\right)\right] \tag{7}$$

For maximum concentration, $\frac{d\gamma}{dx}=0$ gives:

$$\sigma_z=\frac{H_e}{\sqrt{2}} \tag{8}$$

Substituting this value in expression (5):

$$\gamma_{\max}=\frac{0.117Q_p}{\overline{U}\sigma_y\sigma_z} \tag{9}$$

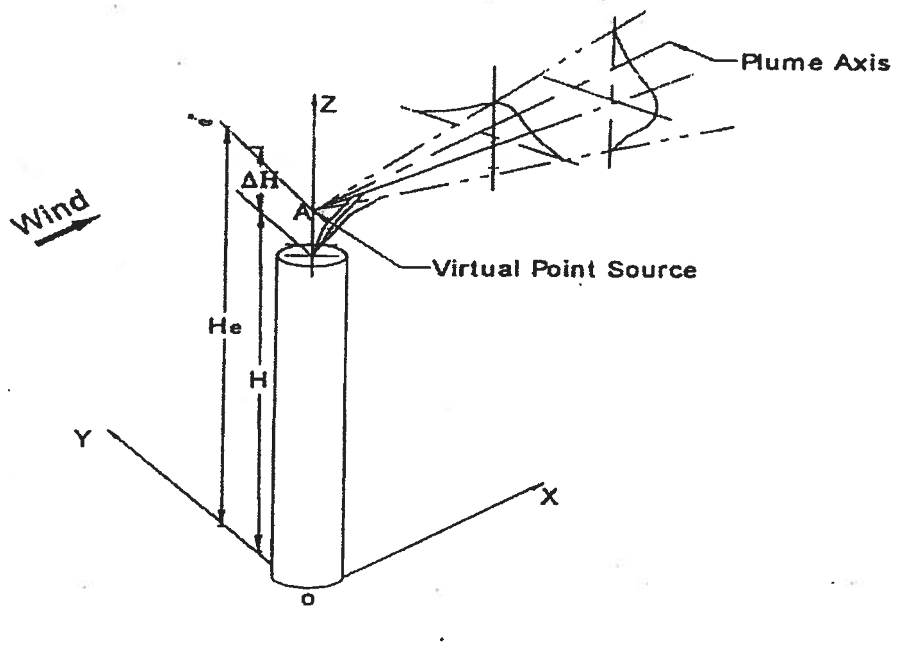

**Figure 1 Gaussian Plume Spread Model**

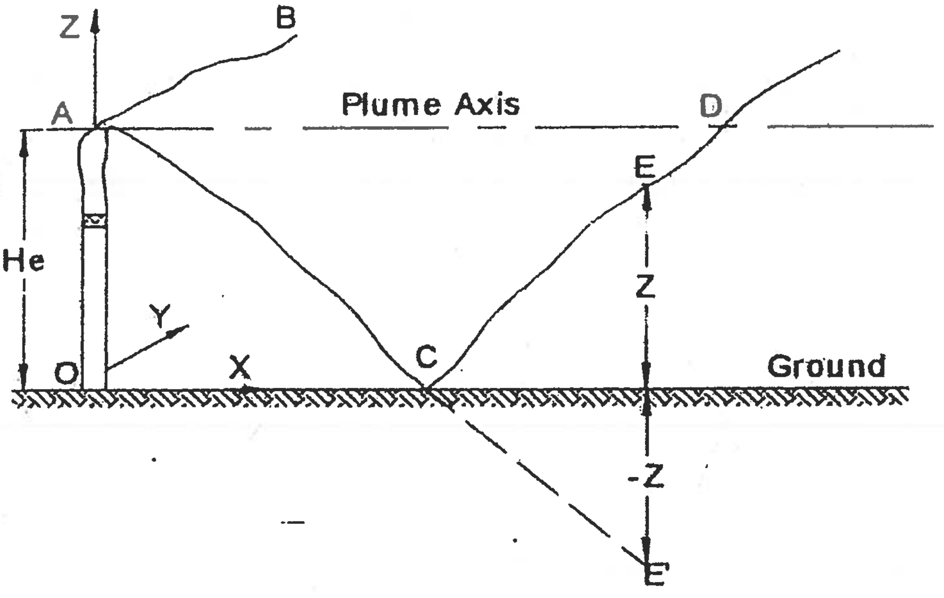

**Figure 2: Effect of Ground Surface on Pollutant Dispersion**

It is observed from above formulation that maximum value of GLC is inversely proportional to wind speed. On the other hand a lower wind speed promotes an increase in buoyancy rise of a plume during day time, hence lowers GLC. This suggests that there is a critical wind speed at which maximum GLC occurs.

The various turbulence and stability classes of atmosphere and the corresponding dispersion conditions are taken into account by dividing 'weather conditions' in stability classes [5]. Three major stability classes are:

- Stable – (low vertical mixing)
- Neutral – (medium vertical mixing)
- Unstable – (strong vertical mixing)

Stability classification is found from Table 1. Night is taken as period from 1 hour before sunset to 1 hour after sun rise [2]. Insolation (in coming solar radiation) and cloudiness data are determined as per Central Pollution Control Board (CPCB) norms as described in Probes/70 [1]. The values of standard deviations $\sigma_y$ and $\sigma_z$ are taken from Tables 2 and 3 for rural and urban conditions. In these Tables, x is downwind distance from source of emission in meters. The area is classified as urban when more than 50% of land inside a circle of 3 km radius around source is considered built up with heavy or medium industrial, commercial or residential units. Standard deviation values depend on many factors, such as atmospheric structure, topography, wind speed, sampling distance from source and sampling time. The uncertainties associated with estimates of $\sigma_y$ and $\sigma_z$ will increase with distance from source.

**Table 1 Stability Classification**

| Surface Wind speed (at 10 m), m/sec | Day Time Insolation | | | Night Time Condition | |
|---|---|---|---|---|---|
| | Strong | Moderate | Slight | Thin overcast or ≥4/8 low cloud cover | ≤3/8 cloud cover |
| < 2 | A | A-B | B | - | - |
| 2-3 | A-B | B | C | E | F |
| 3-5 | B | B-C | C | D | E |
| 5-6 | C | C-D | D | D | D |
| > 6 | C | D | D | D | D |

A=extremely unstable, B=moderately unstable, C=slightly unstable, D=neutral, E=slightly stable, F=stable

**Table 2 Rural Conditions (100 m < x < 10000 m)**

| Atmospheric Stability class | $\sigma_y$ (m) | $\sigma_z$ (m) |
|---|---|---|
| Extremely unstable | $0.22x(1+0.0001x)^{-0.5}$ | $0.20x$ |
| Moderately unstable | $0.16x(1+0.0001x)^{-0.5}$ | $0.12x$ |
| Slightly unstable | $0.11x(1+0.0001x)^{-0.5}$ | $0.08x(1+0.0002x)^{-0.5}$ |
| Neutral | $0.08x(1+0.0001x)^{-0.5}$ | $0.06x(1+0.0015x)^{-0.5}$ |
| Slightly stable | $0.06x(1+0.0001x)^{-0.5}$ | $0.03x(1+0.0003x)^{-1}$ |
| Stable | $0.04x(1+0.0001x)^{-0.5}$ | $0.016x(1+0.0003x)^{-1}$ |

**Table 3 Urban Conditions (100 m < x < 10000 m)**

| Atmospheric Stability class | $\sigma_y$ [m] | $\sigma_z$ [m] |
|---|---|---|
| Extremely unstable and Moderately unstable | $0.32x(1+0.0004x)^{-0.5}$ | $0.24x(1+0.001x)^{-0.5}$ |
| Slightly unstable | $0.22x(1+0.0004x)^{-0.5}$ | $0.20x$ |
| Neutral | $0.16x(1+0.0004x)^{-0.5}$ | $0.14x(1+0.0003x)^{-0.5}$ |
| Slightly stable and Stable | $0.11x(1+0.0004x)^{-0.5}$ | $0.08x(1+0.0015x)^{-0.5}$ |

## 2. Estimation of Plume Rise

After leaving chimney, gases rise in atmosphere due to exit momentum and buoyancy forces derived from temperature differential between the effluent and its surrounding. Since GLC of effluents from an elevated point source depends on the inverse square of effective stack height, amount of plume rise obtained is an important factor in reducing GLC of pollutants. The behavior of a plume is affected by a number of parameters, including the initial source conditions (i.e. exit velocity and difference between plume temperature and that of air) and wind speed [6, 7 and 8]. Table 4 presents a summary of the available plume rise formulae expressed in the form

$$\Delta h = \frac{Ex^b}{u^a} \tag{10}$$

For stable momentum plume rise, following formula is used:

$$\Delta h = \frac{1.5}{\sqrt[6]{S}} \sqrt[3]{v_s^2 \, d^2 \frac{T_a}{4T_s u}} \tag{11}$$

The variables in Table 4 expressed in consistent set of units are:

d = stack diameter, m
$T_a$ = ambient air temperature at stack height, K
$T_s$ = flue gas exit temperature, K
$v_s$ = Flue gas exit velocity, m/sec
F = buoyancy flux parameter, given by:
$F = g d^2 v_s (T_s - T_a)/4T_s$, m$^4$/sec$^2$

The Briggs stability parameter S is given by:

$$S = (g\partial\theta/\partial z)/T_a \text{, sec}^{-2} \tag{12}$$

**Table 4 Summary of Plume Formulae [1], eq. (10)**

| Reference | Atmospheric Stability | a | b | E | Conditions |
|---|---|---|---|---|---|
| Buoyant Plume | | | | | |
| Briggs | Neutral and Unstable | 1 | 2/3 | $1.6F^{1/3}$ | $F < 55$, $x < 49F^{5/8}$ |
| | Stable | 1 | 0 | $21.4\ F^{3/4}$ | $F < 55$, $x \geq 49F^{5/8}$ |
| | | 1 | 2/3 | $1.6F^{1/3}$ | $F \geq 55$, $x < 119F^{2/5}$ |
| | | 1 | 0 | $38.7\ F^{3/5}$ | $F \geq 55$, $x > 119F^{2/5}$ |
| | | 1/3 | 0 | $2.4(F/S)^{1/3}$ | |
| | | 0 | 0 | $5F^{1/4}\ S^{-3/8}$ | |
| | | 1 | 2/3 | $1.6F^{1/3}$ | |
| Momentum Plume | | | | | |
| Briggs | Unstable | 2/3 | 1/3 | $1.44(d\,.\,v_s)^{2/3}$ | $v_s / u \geq 4$ |
| | Neutral | 2/3 | 1/3 | $1.44(d\,.\,v_s)^{2/3}$ | $v_s / u \geq 4$ |
| | | 1 | 0 | $3dv_s$ | $v_s / u \geq 4$ |

Where, $\theta$ is potential temperature, $\frac{\partial\theta}{\partial z}$ is potential temperature gradient and is estimated from:

$$\frac{\partial\theta}{\partial z} = \frac{\partial T}{\partial z} + 0.986 \qquad (13)$$

Where, $\partial T/\partial z$ is temperature change with height above ground surface. Neutral gradient is given by $\partial\theta/\partial z = 0$, stable with $\partial\theta/\partial z$ positive and unstable with $\partial\theta/\partial z$ negative. If appropriate field data are not available to estimate $\partial\theta/\partial z$, Table 5 can be used for finding S.

**Table 5 Atmospheric Stability as a Function of Temperature Gradient**

| Atmospheric Stability class | Temperature gradient, $\partial T/\partial z$ , ($^0C$ / 100$m$) |
|---|---|
| Extremely unstable | < - 1.9 |
| Moderately unstable | -1.9 to –1.7 |
| Slightly unstable | -1.7 to –1.5 |
| Neutral | -1.5 to –0.5 |
| Slightly stable | -0.5 To 1.5 |
| Stable | > 1.5 |

### 2.1 Accuracy of Estimates

The accuracy of dispersion estimates may vary from ±15% to a large value depending on the complexity of dispersion phenomenon as generated by terrain features, meteorological conditions etc. The expressions stated above adequately deal with simple situations. However, there are cases when many other factors need to be undertaken, such as special terrain features, ground slopes, abnormal meteorological conditions applicable to the site etc. Often, such situations can be approximately accounted for by modifying the basic expressions. The principal assumptions on which dispersion analysis and standard deviation values depend are [9, 10]:

1. Lapse rate is uniform.
2. Wind speed and its turbulent characteristics are uniform over the distance travelled by plume and turning of wind with height is neglected.
3. A flat topography is assumed.
4. The pollutant emission rate is uniform and continuous. It is also assumed that plume rises vertically after emission until it reaches an equilibrium altitude and thereafter travels horizontally.
5. With respect to stationary plume axis, the plume spread profile is taken as Gaussian in both orthogonal transverse directions.
6. None of the effluent is lost from a plume and there is total reflection of gaseous plumes from the ground.
7. Chemical and photochemical reactions along plume path are neglected.

Despite above stated assumptions, GPM is still a basic model for dispersion calculations because of its simplicity in mathematical operations and its consistency with the random nature of atmospheric turbulence.

### 2.2. Outcome of Dispersion Analysis

This model is applied to any stack source emitting pollutants. GLC, max. GLC and plume rise are estimated by above expressions and compared with permissible values. This dispersion model is further used to examine the effect of changing stack height, gas volume, efflux velocity and temperature and / or pollutant emission rate on GLC iteratively. Computer programs based on dispersion model can also be developed to examine these effects.

Above formulation in association with guidelines in CPCB Probes/70 describe the model, type of on-site meteorological data requirements, methods of data collection, default parameter values (when on-site data cannot be collected), methods for determination of atmospheric stability, methods to estimate effective stack height and mixing height. It is advised that recommended guidelines are followed in their totality while conducting environmental impact studies to air environment for the purpose of environmental clearance [1].

## 3 Conclusions

The important conclusions drawn from above formulation are:

- When $\sigma_y/\sigma_z$ is independent of x, maximum GLC along the plume axis occurs at a distance where $\sigma_z = \frac{H_e}{\sqrt{2}}$ which is dependent only on the rate of vertical spread.
- Maximum value of GLC is inversely proportional to wind speed. On the other hand, lower the wind speed, higher the buoyancy rise of a plume and hence lesser GLC. This suggests that there is a critical wind speed at which maximum GLC occurs.
- Amount of plume rise obtained is an important factor in reducing GLC of pollutants. GLC of effluents from an elevated point source depends on the inverse square of effective stack height.

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